Quantifying

Non-Destructive Evaluation

Capability and Reliability

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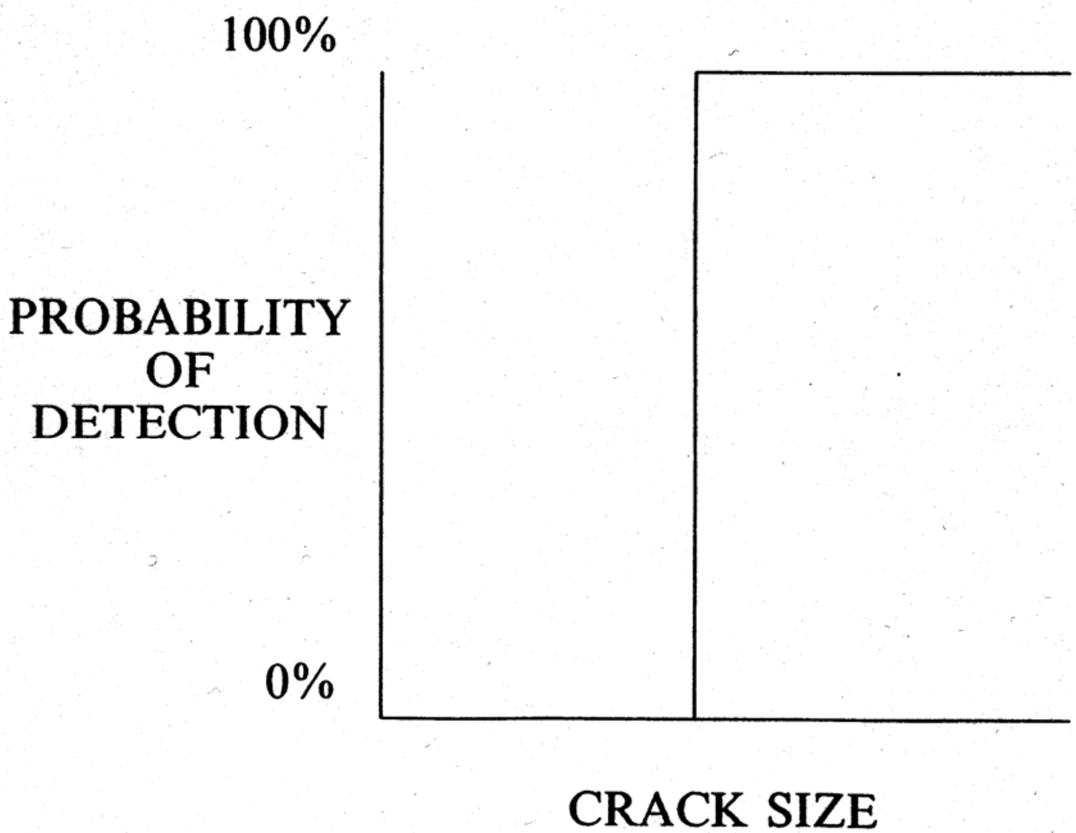
Inspection Basics

NON DESTRUCTIVE EVALUATION

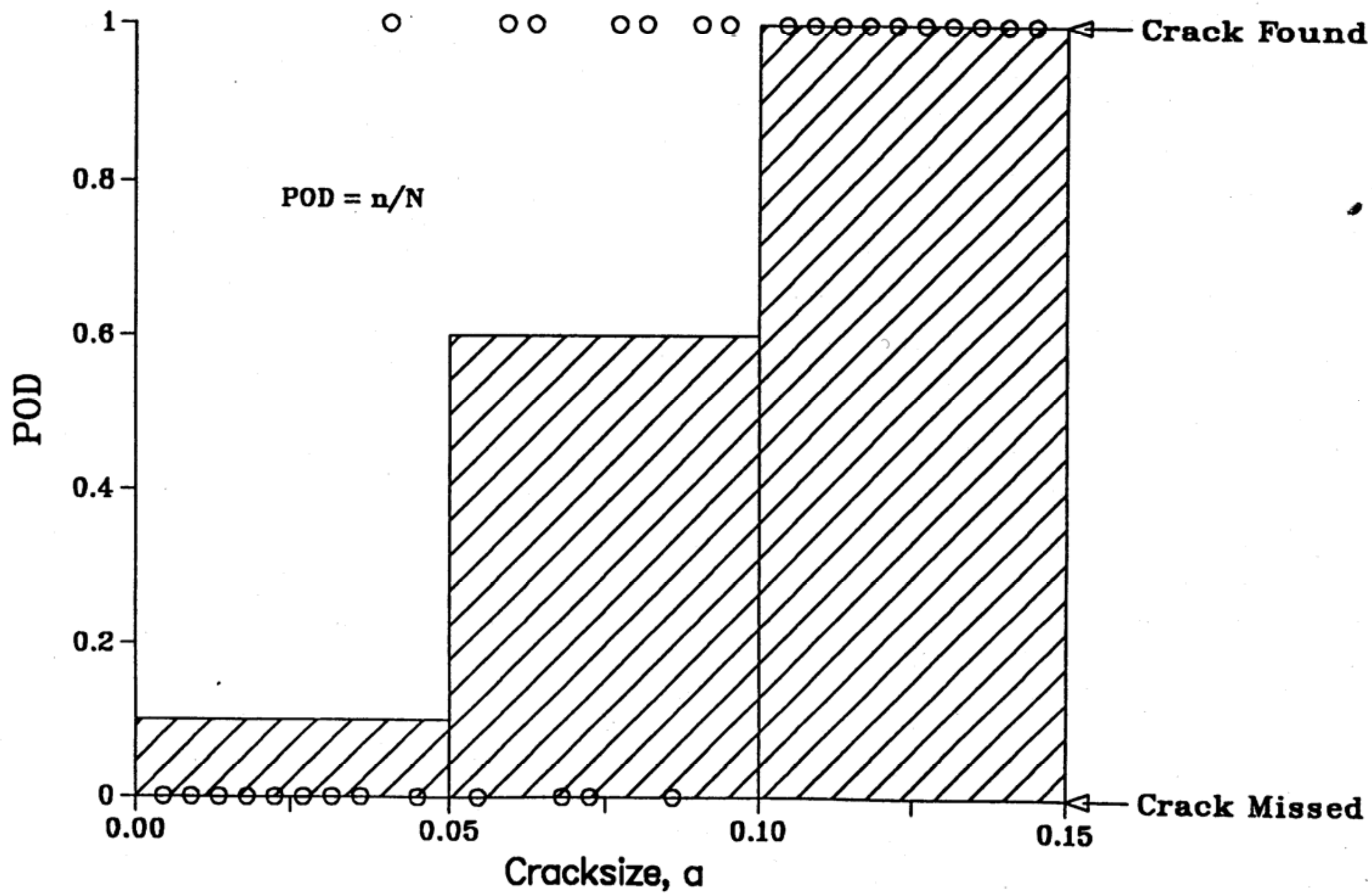


NDE Systems Classification

- ″hit/miss″ systems, which produce only qualitative information as to the presence or absence of a flaw.

- ″a-hat vs. a″ systems, which also provide some quantitative measure of the size of the indicated flaw.

Figure 1
Resolution in POD vs. Resolution in Cracksize



MODELING POD FOR HIT/MISS DATA

$$L(\underset{\sim}{\theta};\ \underset{\sim}{a},\ \underset{\sim}{x}) = \left[\prod_{i=1}^{h} P_i\right]\left[\prod_{j=1}^{n-h} (1 - P_j)\right]$$

where $\underset{\sim}{\theta} = \begin{bmatrix} \mu \\ \sigma \end{bmatrix}$

h = no. of hits

n = total no. of observations

The (log) Logistic Function

is sometimes used to model $POD(a)$.

$$POD(a) = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}}$$

Where:

$x = \log(a)$, and
$(\beta_0, \beta_1)^T$ are model parameters.

The (log) Logistic Function

has interesting properties.

$$g(x) = \ln\left[\frac{POD(a)}{1 - POD(a)} \right]$$

So that ...

$$g(x) = \beta_0 + \beta_1 x$$

<u>Nevertheless,</u> the Normal Hazard function, used to model "a-hat vs. a" data, is also used to model "hit/miss" data, to facilitate comparisons between the different inspection techniques.

The *POD*(*a*) Function

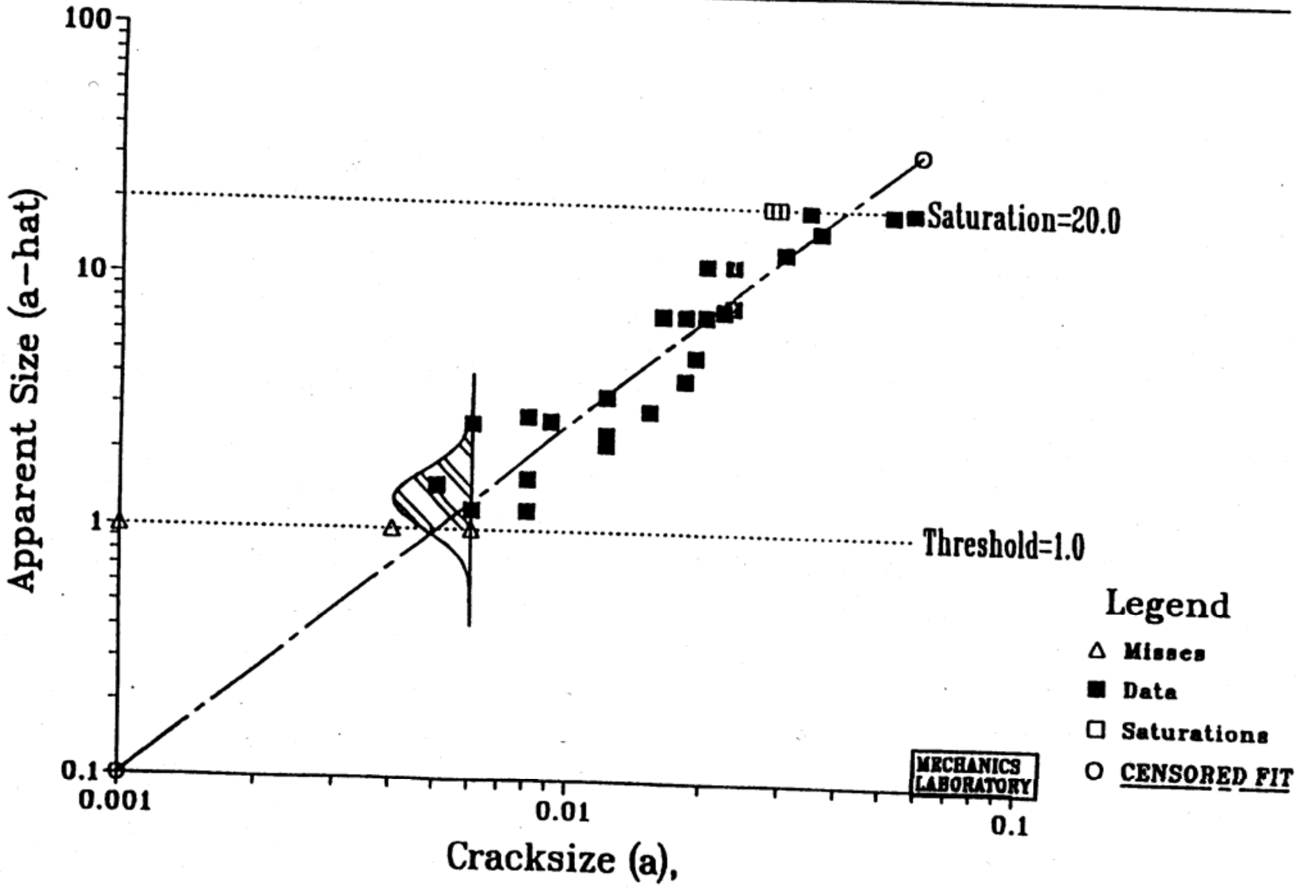
Relates the Probability of Detection to Cracksize, *a*

$$POD(a) = 1 - Q\left[\frac{x - \mu}{\sigma}\right]$$

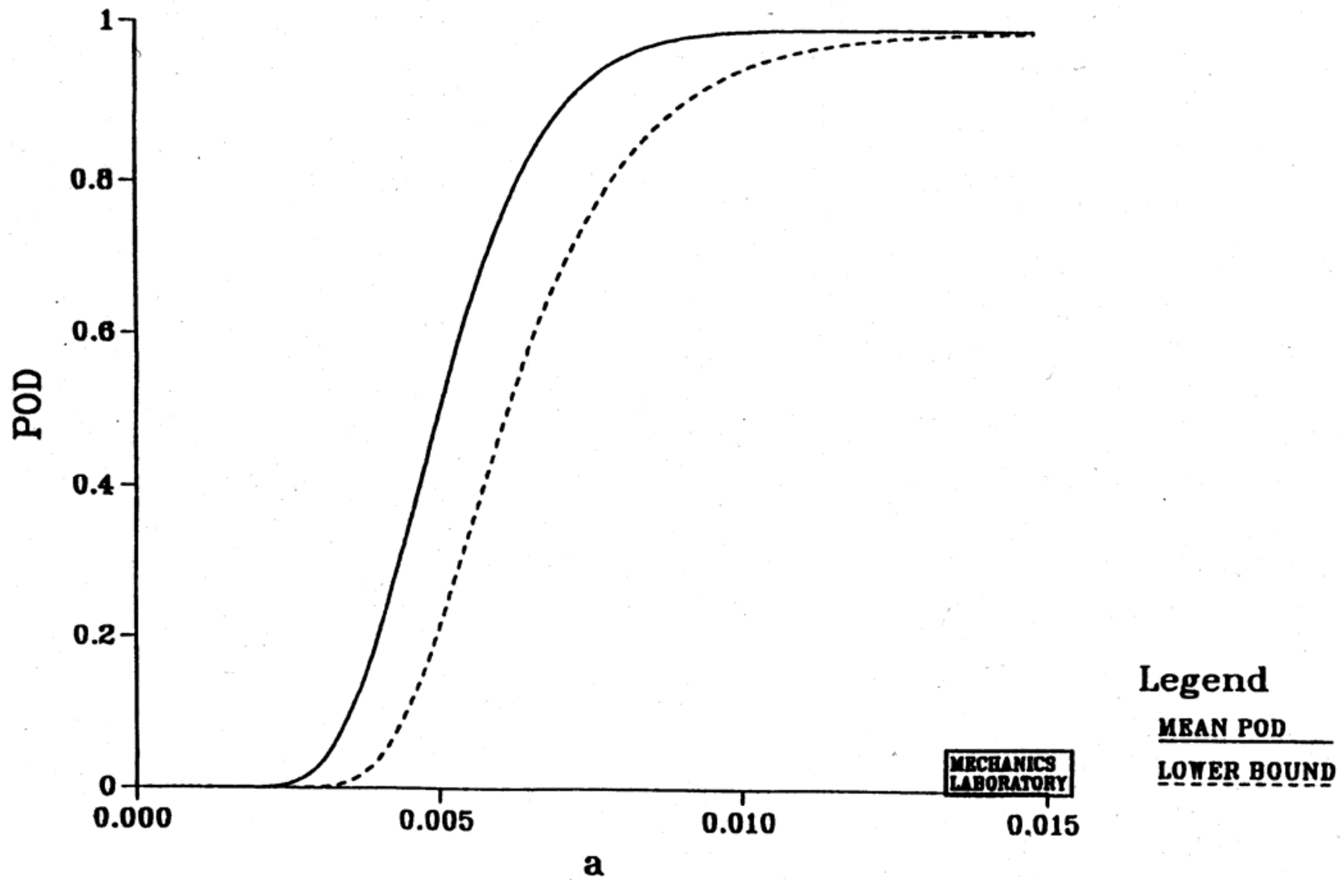
Where:

Q is the Normal Survivor Function,
$x = \log(a)$, and
$(\mu, \sigma)^T$ are model parameters.

APPARENT CRACKSIZE IS RELATED TO ACTUAL SIZE



$\hat{a}$ vs. a CAN BE TRANSLATED TO POD(a)



Elementary Statistical Concepts for NDE

Statistics for NDE Engineers

(Secrets that Statisticians don't want Engineers to know)

Statistics Overview (continued)

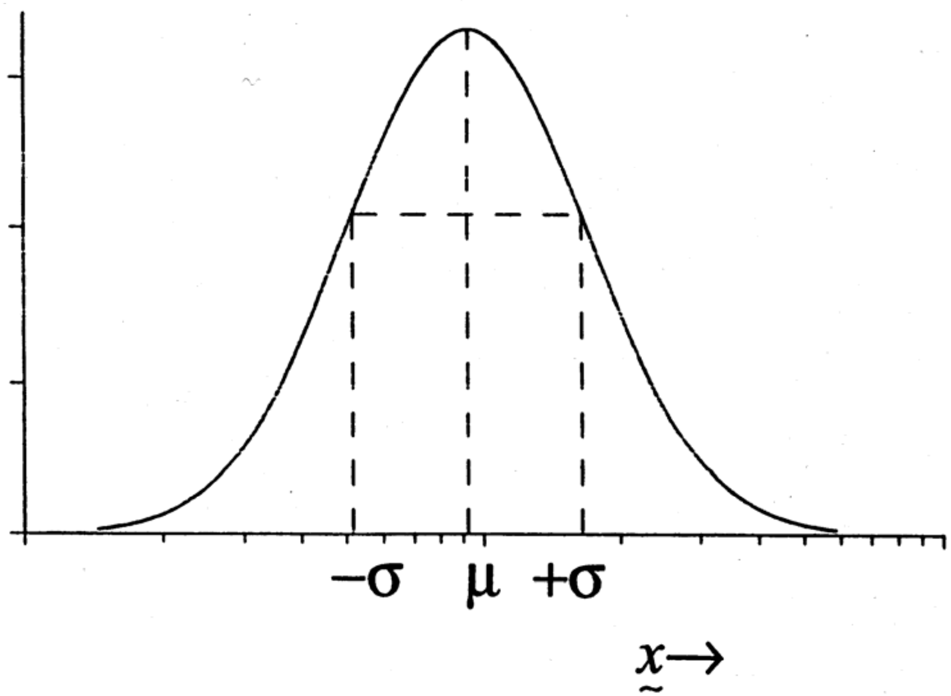
A "statistic" is a function of the data alone.

eg: $\bar{x} = f(x_1, x_2, \ldots, x_n)$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

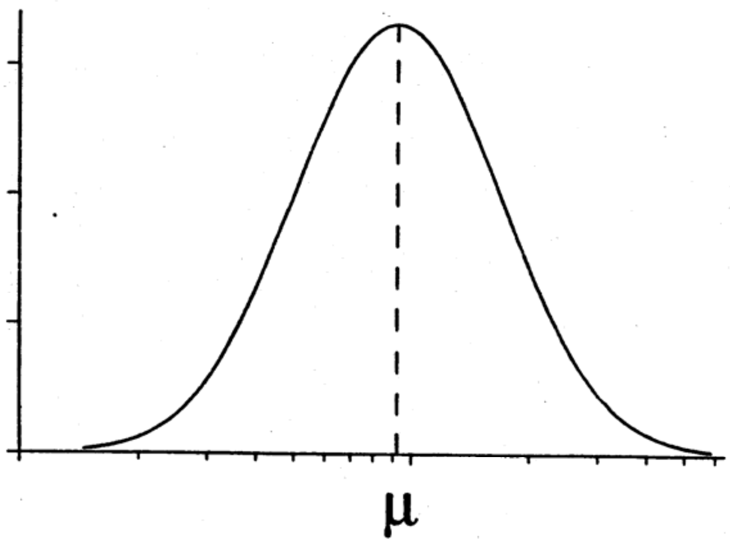
Statistics Overview (continued)

Engineers are familiar with probability distributions of physical characteristics.



Secret #1

Statisticians work with probability distributions of population parameters*



(actually, "estimates" of population parameters ie: statistics)

A "Population Parameter" ...

- describes the underlying behavior of the data.
  eg: (μ, σ^2 for Normal Distribution)

- is fixed (it has no distribution).

A "Population Parameter" ...

- is not a function of the observations, but can be estimated from them.

- these "Parameter Estimates" are functions of the data. (more on this later)

"Probability" is a statement about frequency of occurrence.

Here's the Plan:

We'll discuss a simple univariate example, then generalize to the multivariate case used to quantify NDE performance.

A Review of Probability

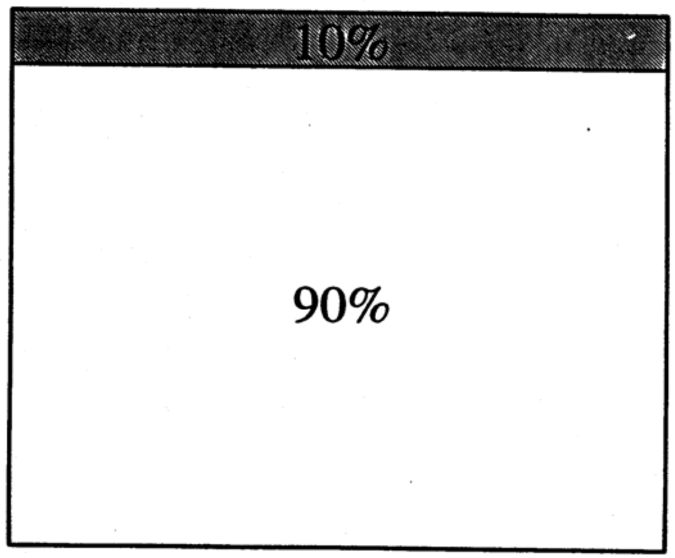
for people who hate Statistics:

$$P(x) = n_x/N$$

where: n_x is number of "successes,"
N is total number of opportunities,
and a "success" has trait x.

Probability Review, cont.

$P(x)$ is the relative frequency of occurrence of trait x.
(eg: X = fatigue life of x cycles)

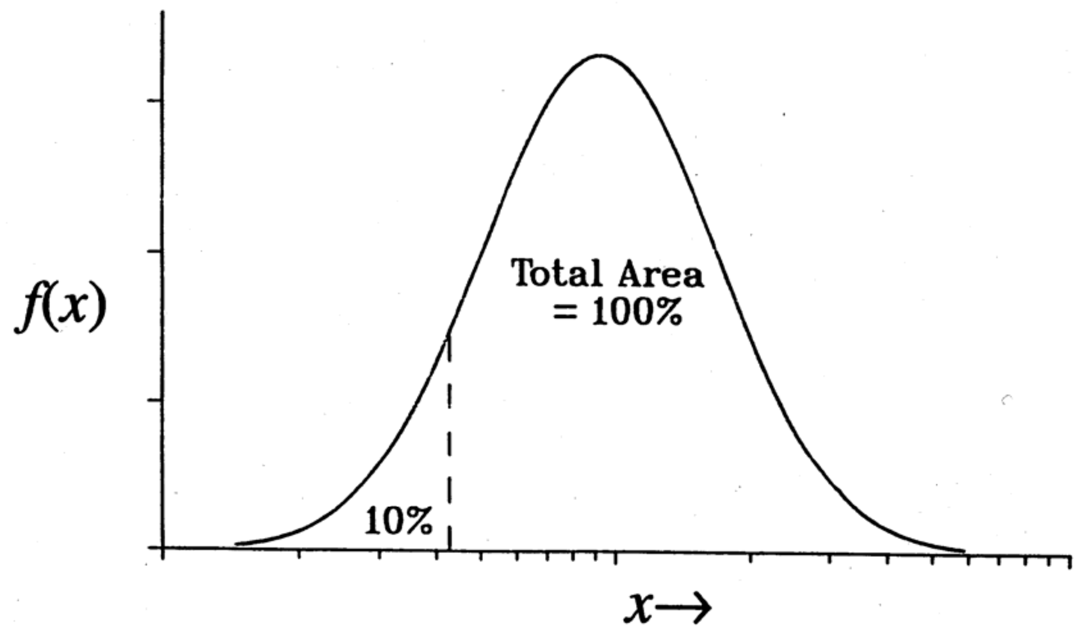


$$p = \frac{10}{90+10} = 10\%$$

VENN diagram

Probability Review, cont.

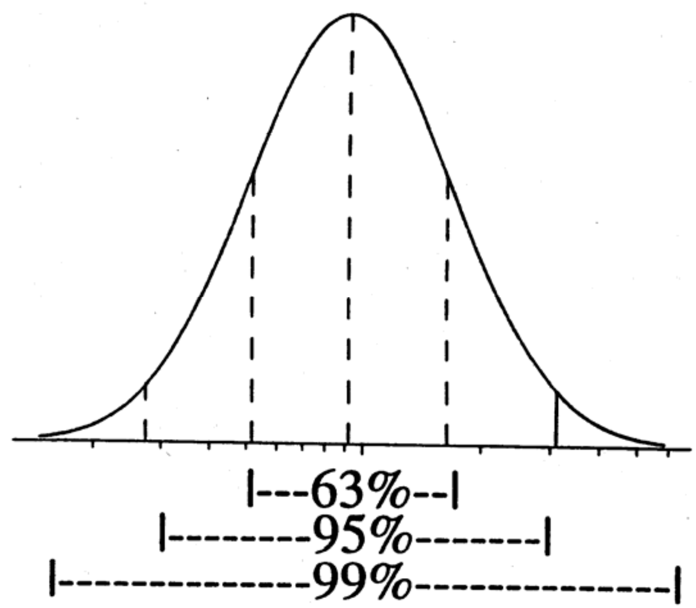
A "Probability Distribution" describes the relationship between x and the relative frequency of x, $f(x)$.



Probability Review, cont.

example:

If $X \sim N(\mu, \sigma^2)$



Then,

$$P(\mu - 1\sigma < x < \mu + 1\sigma) \approx 63\%$$
$$P(\mu - 2\sigma < x < \mu + 2\sigma) \approx 95\%$$
$$P(\mu - 3\sigma < x < \mu + 3\sigma) \approx 99\%$$

Probability Review, Summary

Population parameters (eg: μ, σ^2) tell how often to expect a given value of x.

$$\text{Probability} = P(\underset{\sim}{x} \mid \underset{\sim}{\theta})$$

$$\text{eg: } \underset{\sim}{\theta} = \begin{bmatrix} \mu \\ \sigma \end{bmatrix}$$

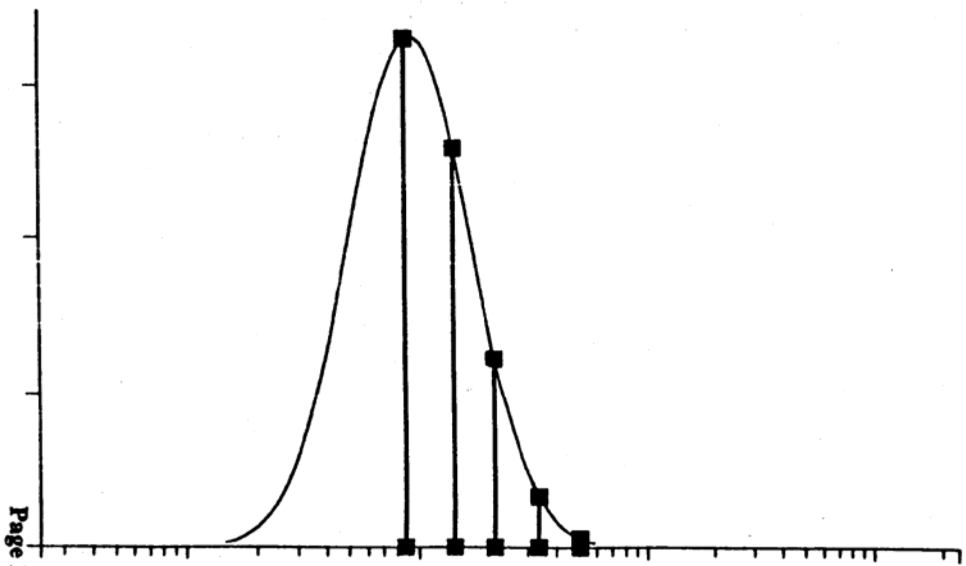
Secret #2

The "Likelihood Function" permits "estimating" Population Parameters from the behavior of the data.

$$\text{Likelihood} = L(\underset{\sim}{\theta} \mid \underset{\sim}{x})$$

Likelihood

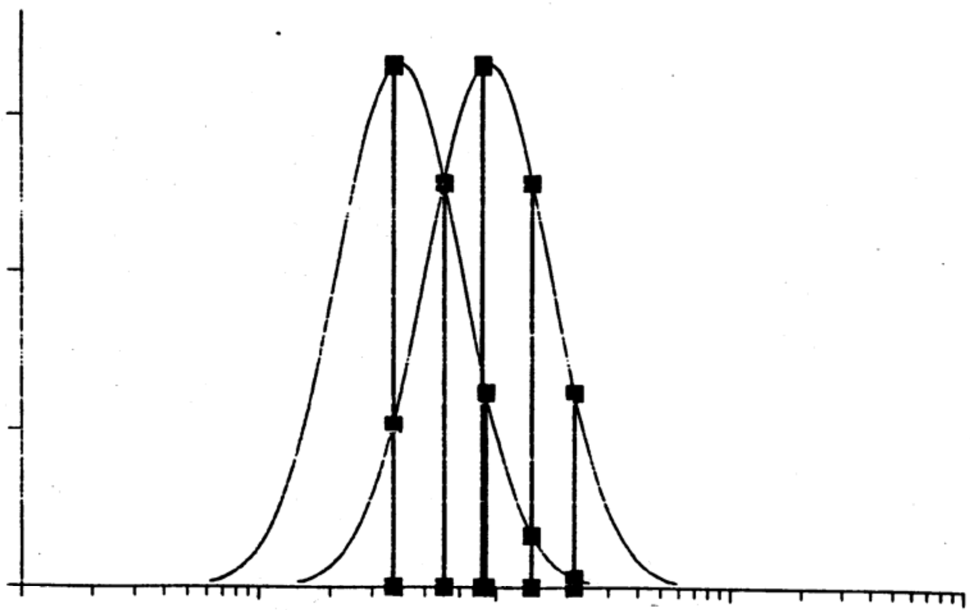
describes the behavior (likelihood) of the population parameter estimates, given the data.



ie: the parameter estimates are a function of the observations

Likelihood

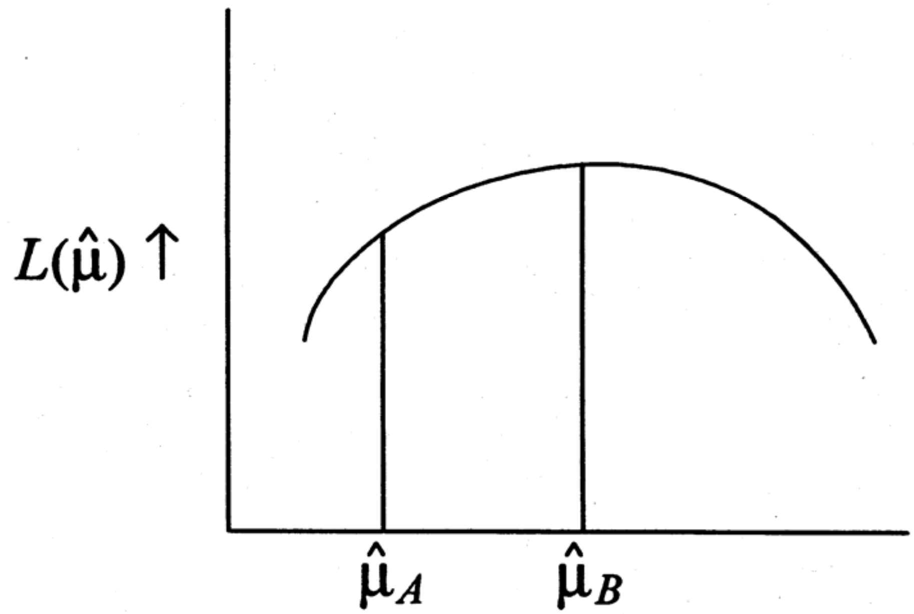
describes the behavior (likelihood) of the population parameter estimates, <u>given</u> the data.



ie: the parameter <u>estimates</u> are a function of the observations

Likelihood has a maximum

(for well-behaved situations)



Secret #3

MLEs (Maximum Likelihood Estimators) are Normally Distributed.

(see example)

MLE Example:

- The sample mean, $\bar{x}$, is a mle and has a Normal Distribution with mean μ and variance σ^2/n, where n is the sample size.

- For large samples this is true even when the population distribution is NOT-Normal.

Secret #3, comment

It is the population parameter *estimates* (not the population values) who's normal behavior can be exploited (for example, to place "confidence-limits" on *POD(a)*).

Statistics Secret #3, details

MLEs are asymptotically Normal with mean $\underset{\sim}{\mu}$[1] and variance-covariance matrix determined [2] from the Likelihood function.

notes:

[1] $\underset{\sim}{\mu}$ = population mean vector

[2] $\underset{\sim}{V} = \left[\frac{\partial^2 L}{\partial \theta^2}\right]^{-1}$

Using Secret #3

So what?

We can estimate *POD(a)* behavior from the experimental data to provide a mean *POD(a)* relationship and construct confidence limits.

POD(a) can be described quantitatively