

# **Transfer Function Model Assisted Probability of Detection for Lap Joint Multi Site Damage Detection**

**Quantitative Nondestructive Evaluation  
Conference 2011**

**July 18-22, 2011**

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# MAPOD for WFD Detection Outline

- **Transfer function approach based on POD model parameters**
- **Unique validation because of having samples from Q4 – real defects characterized in retired real airplane structure**
- **Specimens and basic POD results from inspections**
- **Simple Transfer model**
- **First Approach – linear regression verified using bootstrapped parameter estimates**
- **Second Approach – multivariate regression from bootstrapped hit/miss data**
- **Results & Conclusions**

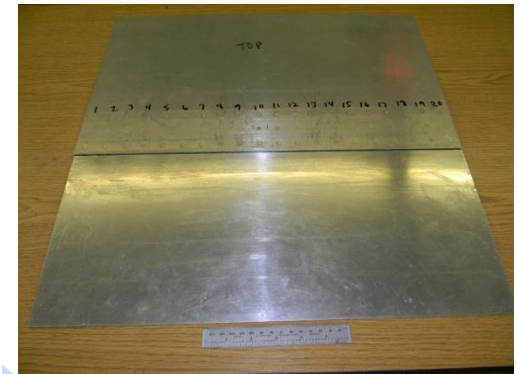
# Basic Specimen Setup

- 4 Inspectors (varying experience levels)
- 4 Sets of POD Specimen Panels



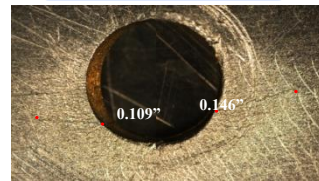
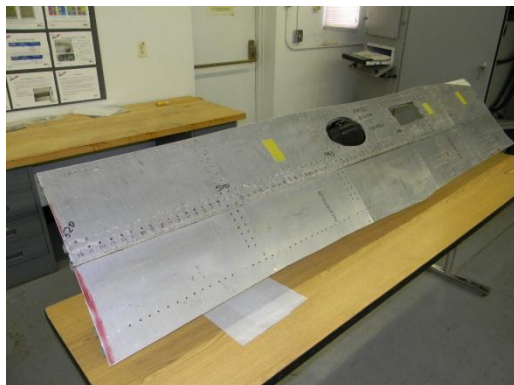
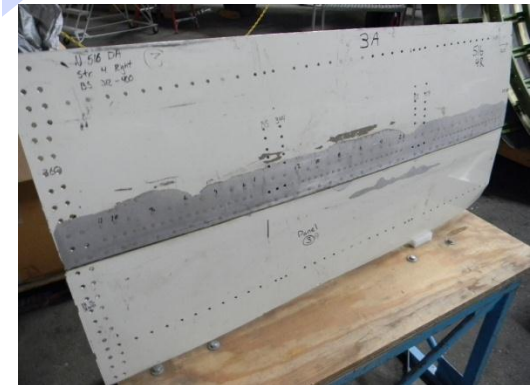
Quad 1  
.039-.141 flaw distribution  
EDM Flaws,  
Simulated  
Structure

Quad 2  
.035-.199 flaw distribution  
Engineered  
Cracks,  
Simulated  
Structure



Quad 3  
.010-.400 flaw distribution  
EDM Flaws,  
Real Structure

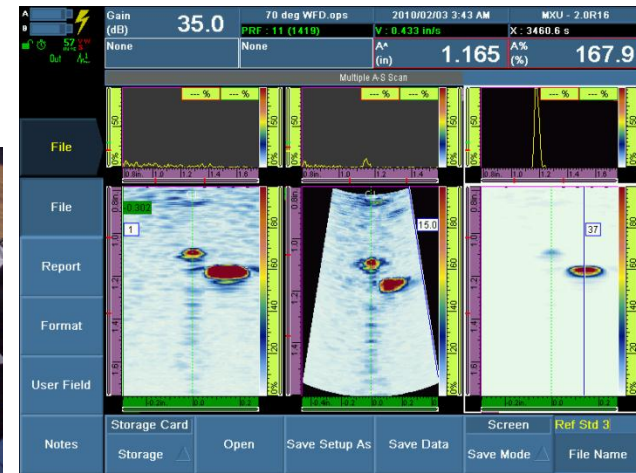
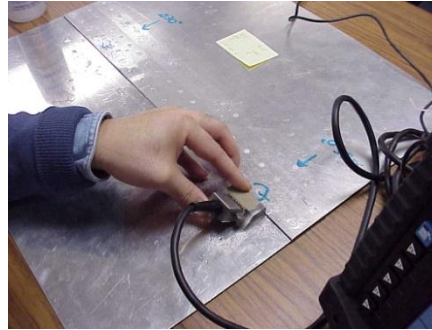
Quad 4  
.019-.274 flaw distribution  
Real Cracks,  
Real Structure  
Paint



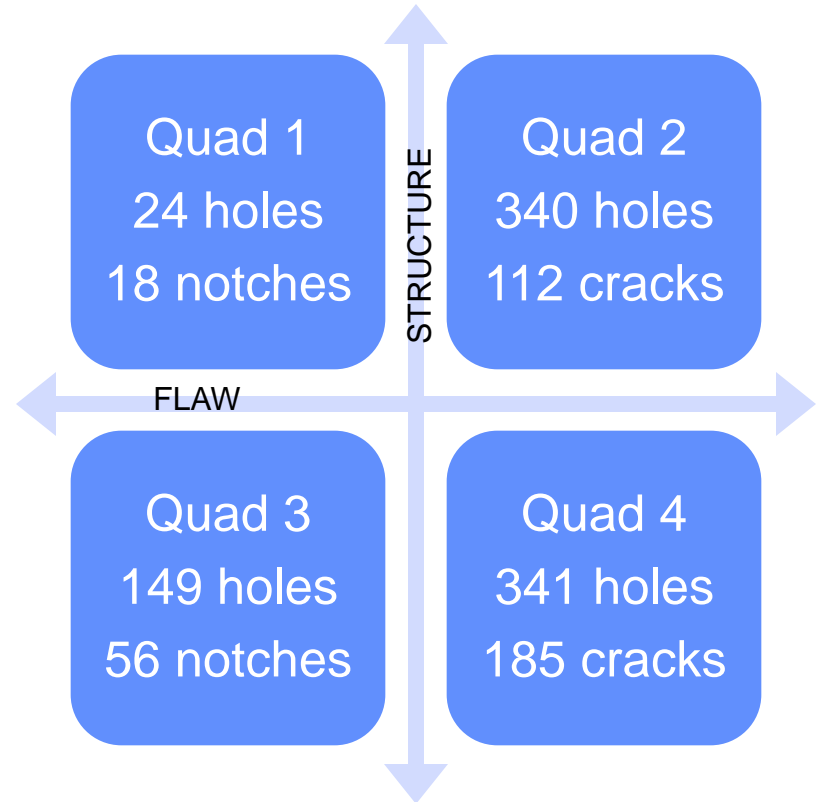
This work was supported by the FAA, William J. Hughes Technical Center, Atlantic City International Airport, New Jersey.

# Inspection

- Method used: Ultrasonic Linear Array Shear Wave on Omniscan Instrument, 10L64 probe (10MHz, 64 element) with 70° wedge



- Each hole was inspected for flaws on the left and on the right sides, providing 2 detection opportunities per hole, each treated independently.
- Quad 4 panels included holes with multiple flaws on one or both sides, and a multi-flaw model was used
- All defect lengths are inches.





## 2 Parameter Probit Model for POD Hit/Miss Data in each Quadrant

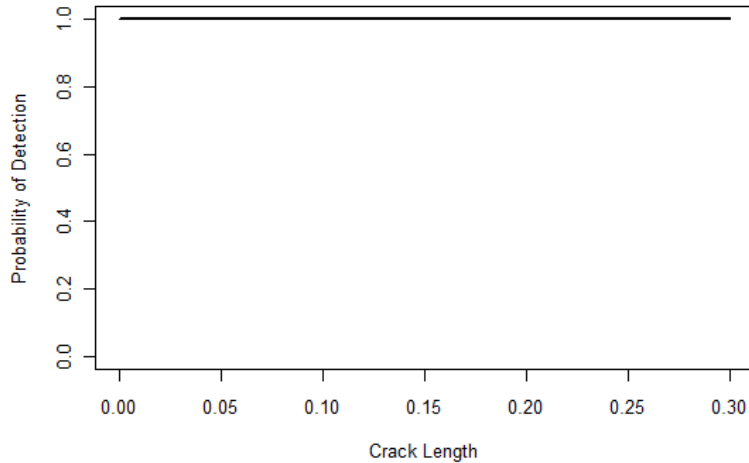
$$\begin{aligned}\text{POD} &= \Phi(\text{Intercept} + \text{Slope} * \ln(\text{length})) \\ &= \Phi(c + d * \ln(\text{length}))\end{aligned}$$

where  $\Phi$  is the cumulative normal distribution

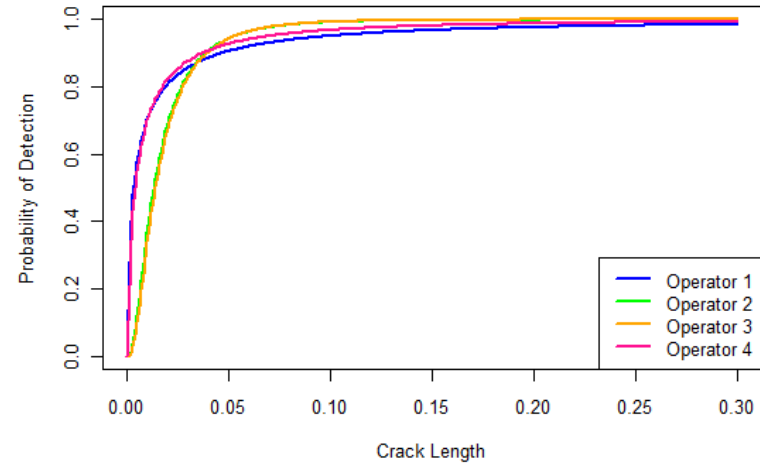
$$\begin{aligned}\text{POD}_{0.90} &= \exp\{(\Phi^{-1}(0.90) - \text{Intercept})/\text{Slope}\} \\ &= \text{smallest crack length for which POD is} \\ &\quad \text{at least 90\%}\end{aligned}$$

# POD curves by Quadrant

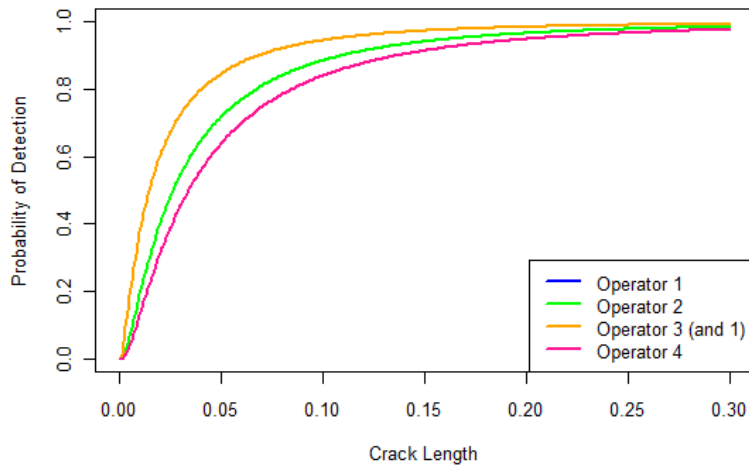
Quad 1: EDM defects, Simulated Structure



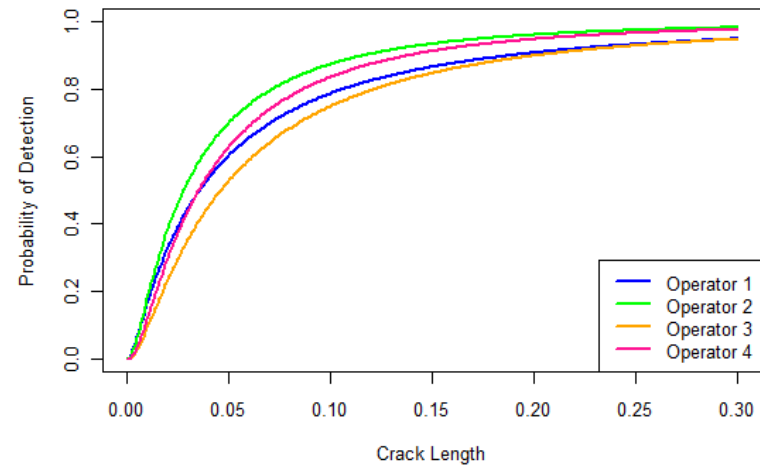
Quad 2: Real Cracks, Simulated Structure



Quad 3: Simulated Cracks, Real Structure



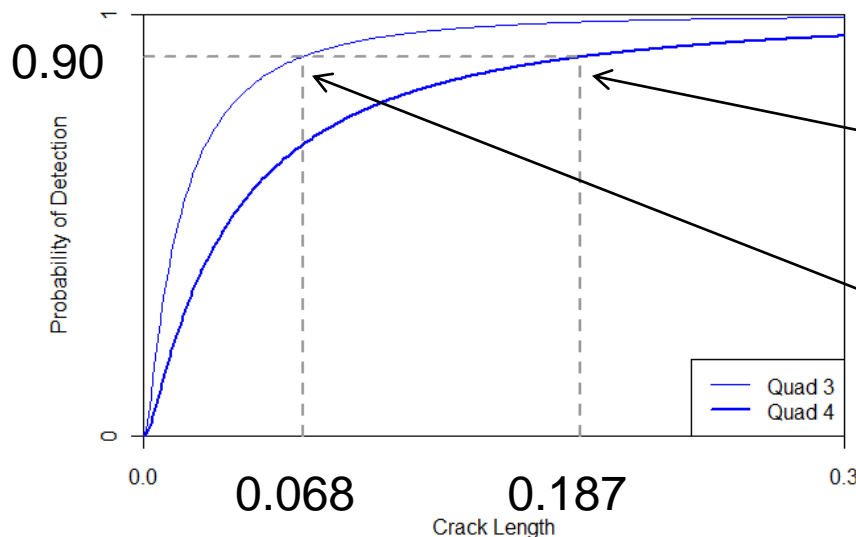
Quad 4: Real Cracks, Real Structure



# Parameters for POD curves

	Quad 2			Quad 3			Quad 4		
Operator	Int $c_2$	Slope $d_2$	POD <sub>0.90</sub>	Int $c_3$	Slope $d_3$	POD <sub>0.90</sub>	Int $c_4$	Slope $d_4$	POD <sub>0.90</sub>
1	2.78	0.49	<b>0.046</b>	3.52	0.83	<b>0.068</b>	2.57	0.77	<b>0.187</b>
2	5.18	1.20	<b>0.039</b>	3.28	0.90	<b>0.109</b>	3.22	0.90	<b>0.116</b>
3	5.32	1.25	<b>0.039</b>	3.52	0.83	<b>0.068</b>	2.68	0.87	<b>0.200</b>
4	3.20	0.58	<b>0.037</b>	3.12	0.92	<b>0.136</b>	3.15	0.94	<b>0.138</b>

Operator 1, Quads 3 & 4



$$\text{POD} = \Phi(2.57 + 0.77 \cdot \ln(\text{length}))$$

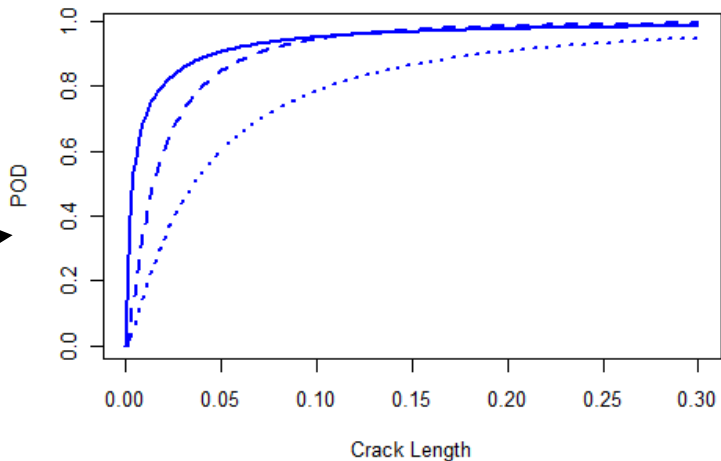
$$0.9 = \Phi(2.57 + 0.77 \cdot \ln(0.187))$$

$$\text{POD} = \Phi(3.52 + 0.83 \cdot \ln(\text{length}))$$

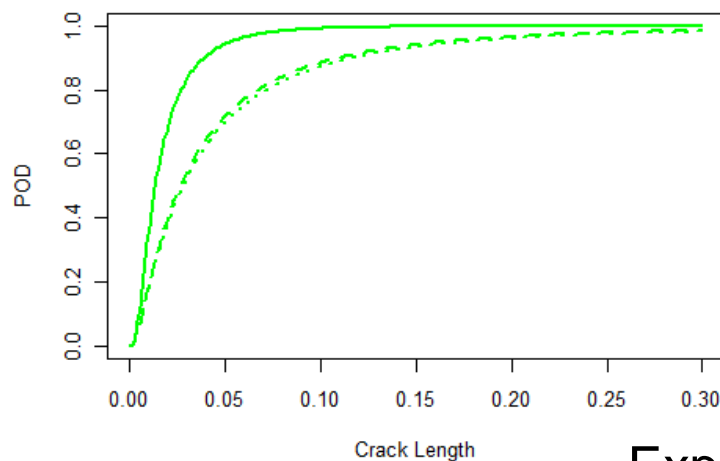
$$0.9 = \Phi(3.52 + 0.83 \cdot \ln(0.068))$$

# POD curves by Operator

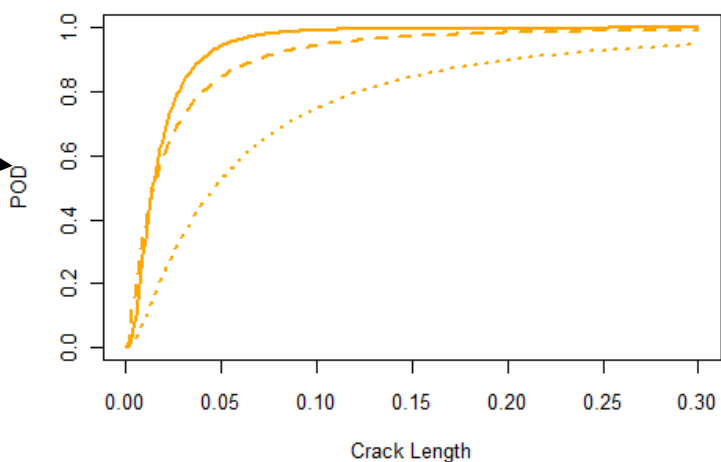
Operator 1



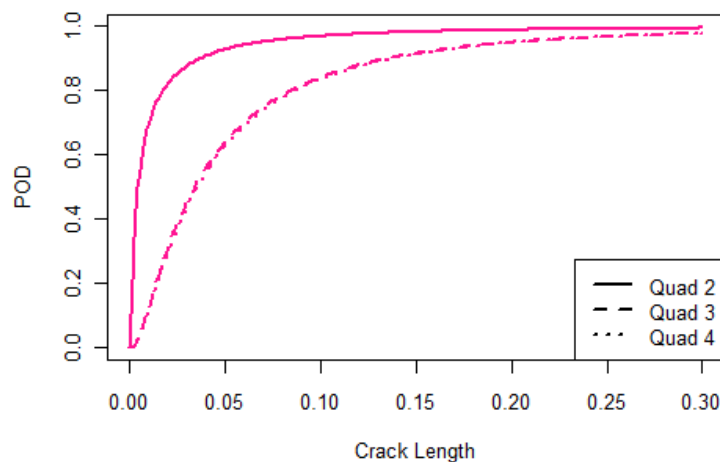
Operator 2



Operator 3



Operator 4



Less Experienced Operators

More Experienced Operators



# False Call Rates

	Quad 2			Quad 3			Quad 4		
Operator	False calls	Opps	Rate (%)	False calls	Opps	Rate (%)	False calls	Opps	Rate (%)
1	28	568	4.9	12	242	5.0	40	497	8.1
2	25		4.4	1		0.4	29		5.8
3	45		7.9	7		2.9	56		11.3
4	26		4.6	2		0.8	43		8.7



# Model Assisted POD Generation

- Ideally, would like to estimate slope and intercept parameters for Q4 PODs from those in Q1, Q2, and Q3
  - Q1 provides minimal information for fitting
  - too few operators to estimate Q4 parameters jointly from Q2 and Q3 parameters, which would be best option
- First look at POD parameter estimates and their relationships
  - fairly strong linear relationships between c & d within a quadrant
  - some relationship between Q3 and Q4 parameters
  - negligible relationship between Q2 and Q4 parameters
- With data from 4 operators, we can try simple models
  - predict  $c_4$  from  $c_2$ ,  $c_3$  & predict  $d_4$  from  $d_2$ ,  $d_3$  using linear regression
    - $c_4 = \beta_0 + \beta_1 * c_2 + \beta_2 * c_3 + \epsilon$
    - $d_4 = \beta_0 + \beta_1 * d_2 + \beta_2 * d_3 + \epsilon$
  - other models are possible



# Use Simulation to Demonstrate Potential MAPOD Methodologies

- Would like to improve on the simple strategies above, but need more data
- First Approach
  - Simulate many  $(c_2, c_3, d_2, d_3)$ 's assuming a joint normal distribution with observed covariance
  - Use independent regressions to estimate  $c_4$  from  $c_2, c_3$ ; and estimate  $d_4$  from  $d_2, d_3$  (best we can do with 4 operators)
  - Compare, for Q4,
    - Predicted PODs generated from regression estimates
    - PODs generated from original 4 operators
- Second Approach
  - Bootstrap hit/miss data for 36 additional operators
  - Generate POD curves and thus  $(c_2, c_3, d_2, d_3, c_4, d_4)$  based on new hit/miss data
  - Use multivariate regression to jointly estimate  $(c_4, d_4)$  from  $(c_2, c_3, d_2, d_3)$  for 40 total operators
  - Compare, for Q4,
    - Predicted PODs generated from multivariate regression estimates for 4 operators
    - PODs generated from original 4 operators

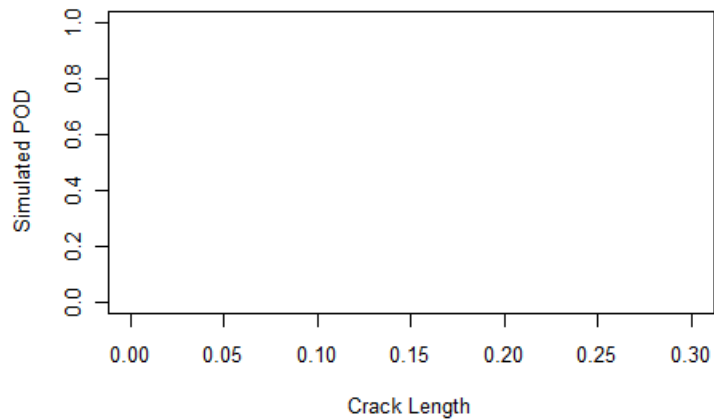


# First Simulation Approach: Generate parameter estimates and predict Q4 c's & d's using independent regressions.

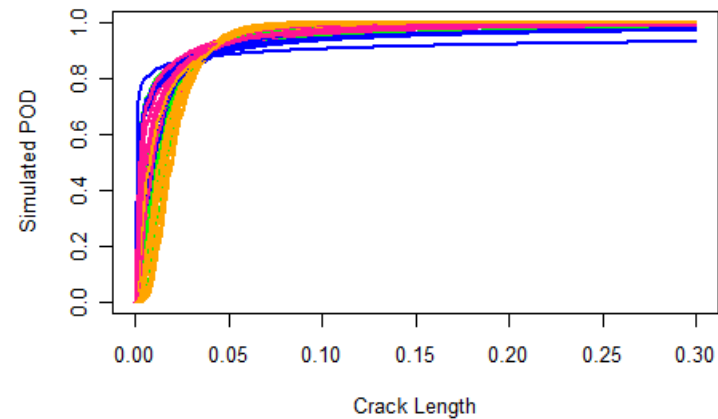
- For each operator and observed  $(c_2, d_2, c_3, d_3)$  quadruple, generate 2500 more quadruples (total of 10,000 quadruples) assuming
  - joint normal distribution
  - mean equal to the observed quadruple value
  - covariance equal to the covariance matrix from the four observed operators
- Use the least squares line from the original operators to predict 10,000 values for  $c_4$  based on  $c_2$  and  $c_3$  and 10,000 values for  $d_4$  based on  $d_2$  and  $d_3$
- Look at resulting 10,000 POD curves for Q4 (sample of 40 curves on next slide)

# First Simulation Approach: A Selection from 10,000 Simulations

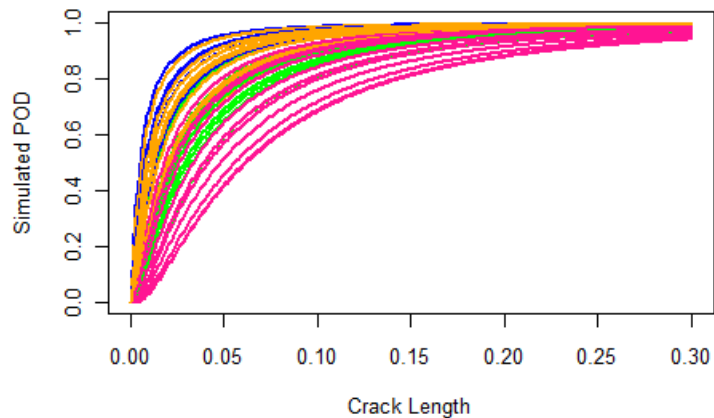
Quad 1: Simulated Flaws, Simulated Structure



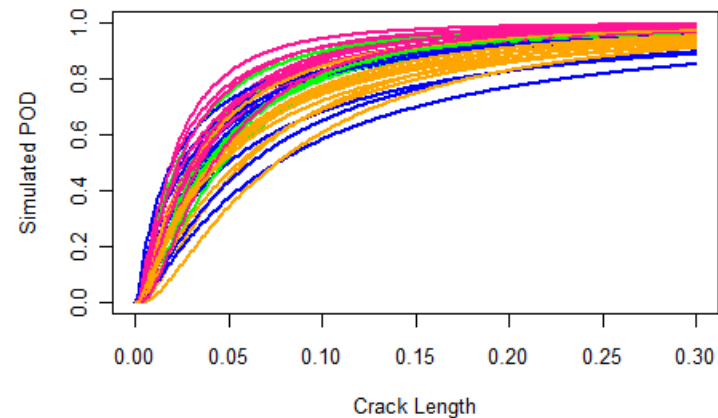
Quad 2: Real Flaws, Simulated Structure



Quad 3: Simulated Flaws, Real Structure



Quad 4: Real Flaws, Real Structure





# POD<sub>0.90</sub> Estimates

## First Simulation Approach

	Quad 2	Quad 3	Quad 4
Median from Observed Operators	0.039	0.088	0.164
Median from Sims (upper 95% CL)	0.039 (0.057)	0.090 (0.184)	0.156 (0.279)

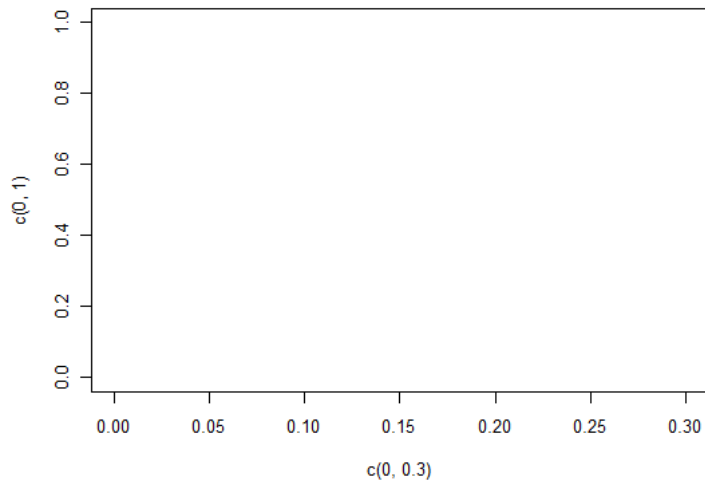
- In Quad 2, the intercept and slope parameter estimates were highly variable for the 4 observed operators (despite the corresponding POD curves being quite similar), causing some simulated Quad 2 POD curves to level off early, or to have negative parameter values
- **Conclusion:** The basic linear regressions used to predict  $c_4$  and  $d_4$  lead to adequate predictions of POD<sub>0.90</sub> in Quad 4... but perhaps we can do better!



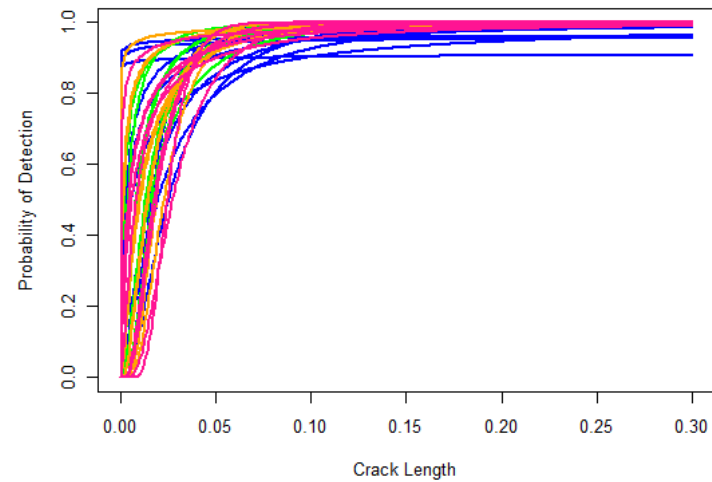
## **Second Simulation Approach: Bootstrap datasets for more operators and generate each POD.**

- Generate POD curves for each operator in Q2, Q3, Q4
- Use the POD curves and random Bernoulli trials to bootstrap hit/miss data for 9 simulated operators based on each of the four observed operators
- Fit POD curves in all quadrants for 40 operators (4 original and 36 bootstrapped), generating  $(c_2, d_2, c_3, d_3, c_4, d_4)$  for each
- Use multivariate regression to jointly estimate  $(c_4, d_4)$  from  $(c_2, d_2, c_3, d_3)$
- Resulting 40 PODs (spaghetti plots) in Q2, Q3, and Q4, on next slide

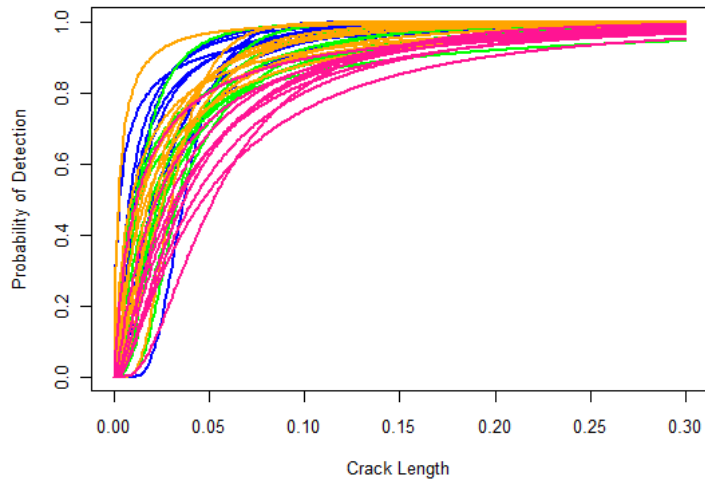
# Bootstrapped POD curves



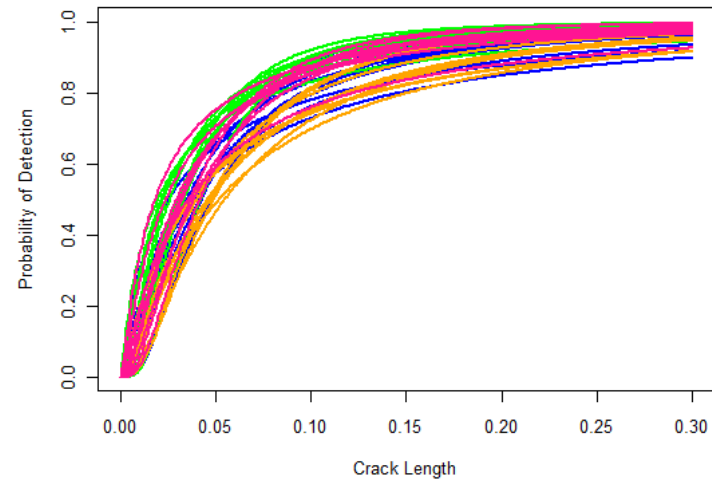
Quad 2: Real Cracks, Simulated Structure



Quad 3: Real Cracks, Simulated Structure



Quad 4: Real Cracks, Real Structure







# POD<sub>0.90</sub> Estimates

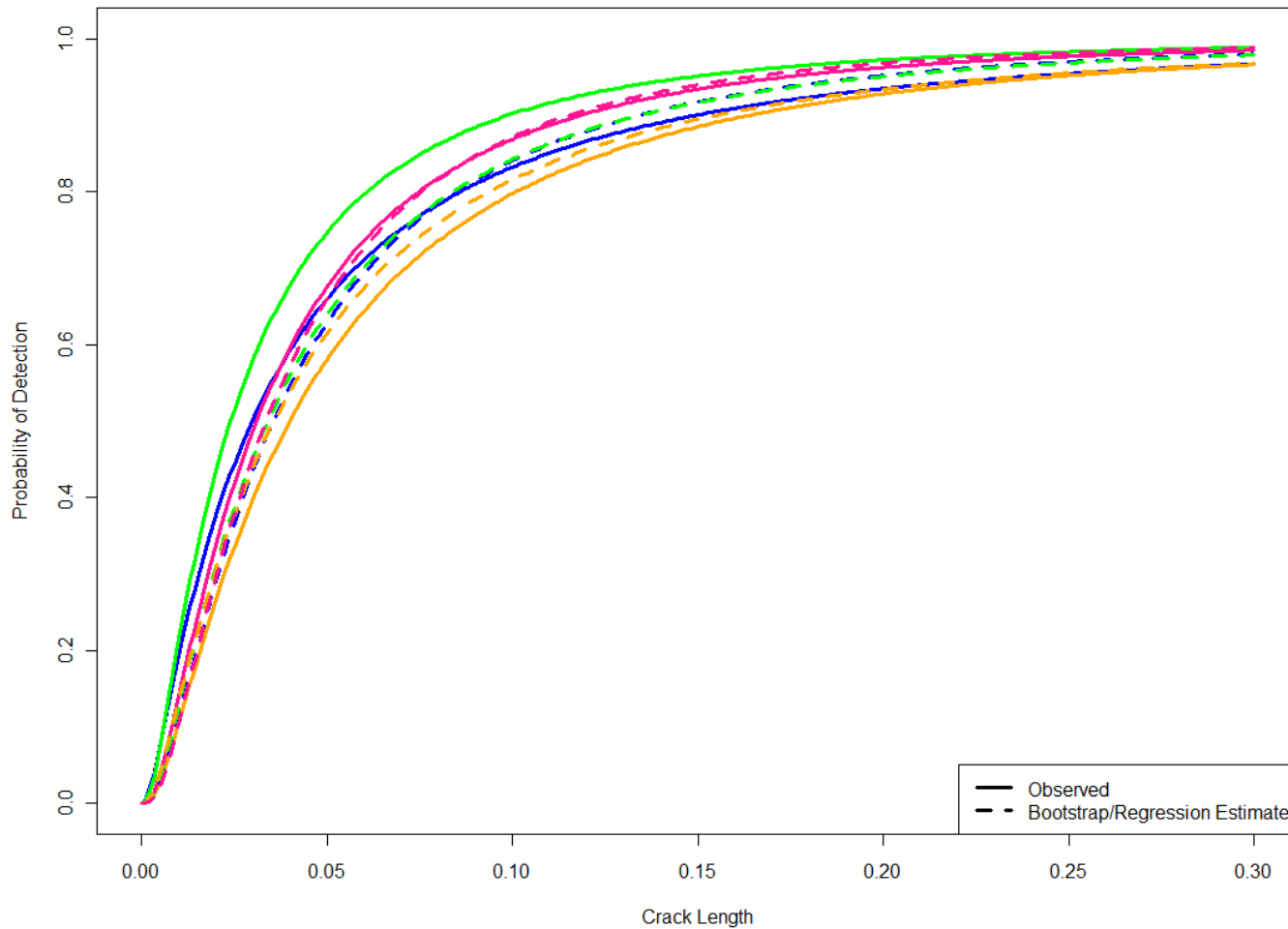
## Second Simulation Approach

	Quad 2	Quad 3	Quad 4
Median from Observed Operators	0.039	0.088	0.162
Median from Bootstrap Operators (Upper 95% CL)	0.039 (0.071)	0.088 (0.156)	0.155 (0.258)

- Bootstrapped datasets did a good job of replicating operator behavior and variability

# Comparison of Observed and Predicted Q4 PODs for 4 Operators (Second Approach)

Quad 4: Real Cracks, Real Structure



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Jersey.



# POD<sub>0.90</sub> Estimates from Original POD and Regression Estimated POD

	Original POD	Regression Estimated POD
Operator 1	0.187	0.134
Operator 2	0.116	0.135
Operator 3	0.200	0.155
Operator 4	0.138	0.116
<b>Average</b>	<b>0.160</b>	<b>0.135</b>

- Multivariate regression estimates for Q4 Bootstrap Operator PODs tended towards anticonservative estimates of POD<sub>0.90</sub>
  - More work is needed to investigate the results of the multivariate regression and other possible correlation structures
  - $\hat{a}$  vs  $a$  POD curves may provide additional information which would allow for a better multivariate regression fit



# Summary

- First simulation strategy offers the potential to generate both  $POD_{0.90}$  and an 95% upper confidence bound on  $POD_{0.90}$  from limited data
  - Still requires *some* data in Q4
  - Somewhat to our surprise, these independent linear regression models from the first example do quite well
- First step of second strategy, bootstrapping to simulate additional operators, appears to have good potential
  - Also requires *some* data in Q4
  - More work (investigation of correlation structure) is needed to determine if the strategy is viable for developing better transfer functions for Q4



# Conclusions & Future Work

- We did not have complete signal strength data readily available, but may pursue  $\hat{a}$  vs  $a$  MAPOD when they are available
- Q3 POD appears to be strong predictor of Q4 POD for some operators in our particular setting
- Q3 carries information to predict Q4, while Q2 appears to carry very little
  - This is good news since Q3 panels (simulated flaws, real structure) are less expensive to produce than Q2 panels (real flaws, simulated structure)
  - Simulated flaws in real structure (Q3) may be more representative of real flaws in real structure (Q4) than real flaws in simulated structure (Q2) because of the influence of the structure
- We know paint attenuates the UT signal, which we suspect is at least part of the reason for the degeneration of POD curves from Q3 to Q4
  - The degree of attenuation is measurable so quantifying that change, and its influence on the predicted POD, is on the agenda
- Ultimate goal is to extend MAPOD methodology to large area composite airplane inspections



# Extra Slides

## Details of Model Fits, Correlation Structure

This work was supported by the FAA, William J. Hughes  
Technical Center,  
Atlantic City International Airport, New Jersey.



# Simple Model for c4 (as function of c2, c3)

Call:

```
lm(formula = c4 ~ c2 + c3, data = q)
```

Residuals:

```
0.04040  0.09013 -0.07623 -0.05430
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	7.8619	1.3504	5.822	0.108
c2	0.0897	0.0603	1.488	0.377
c3	-1.5834	0.4051	-3.909	0.159

Residual standard error: 0.1361 on 1 degrees of freedom

Multiple R-squared: 0.9416,

Adjusted R-squared: 0.8249

F-statistic: 8.065 on 2 and 1 DF, p-value: 0.2416

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# Simple Model for d4 (as function of d2, d3)

Call:

```
lm(formula = d4 ~ d2 + d3, data = q)
```

Residuals:

```
-0.02005 -0.03301  0.02784  0.02522
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.34447	0.59658	-0.577	0.667
d2	0.07047	0.07822	0.901	0.533
d3	1.32049	0.67127	1.967	0.299

Residual standard error: 0.05387 on 1 degrees of freedom

Multiple R-squared: 0.815,

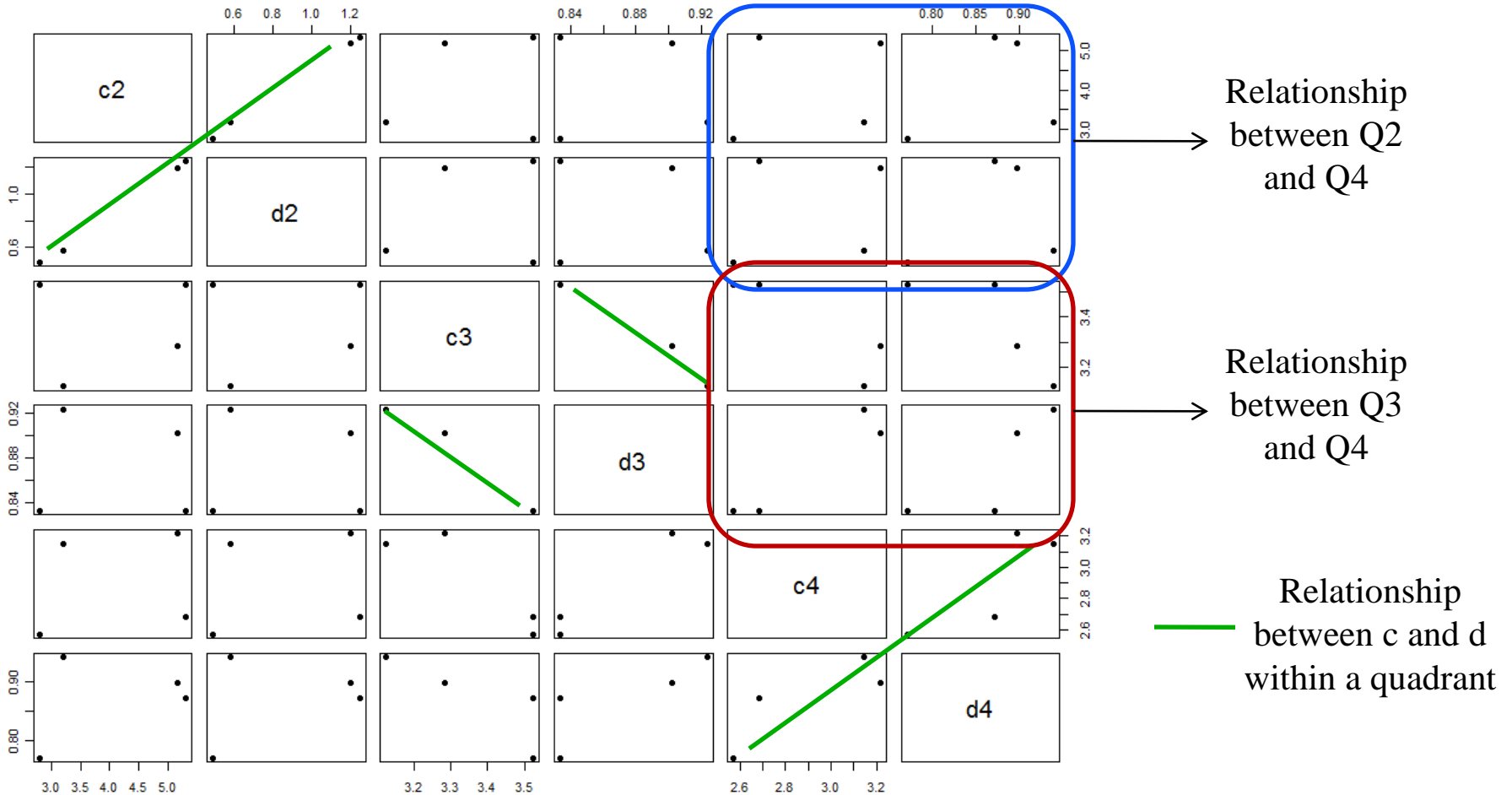
Adjusted R-squared: 0.4451

F-statistic: 2.203 on 2 and 1 DF, p-value: 0.4301

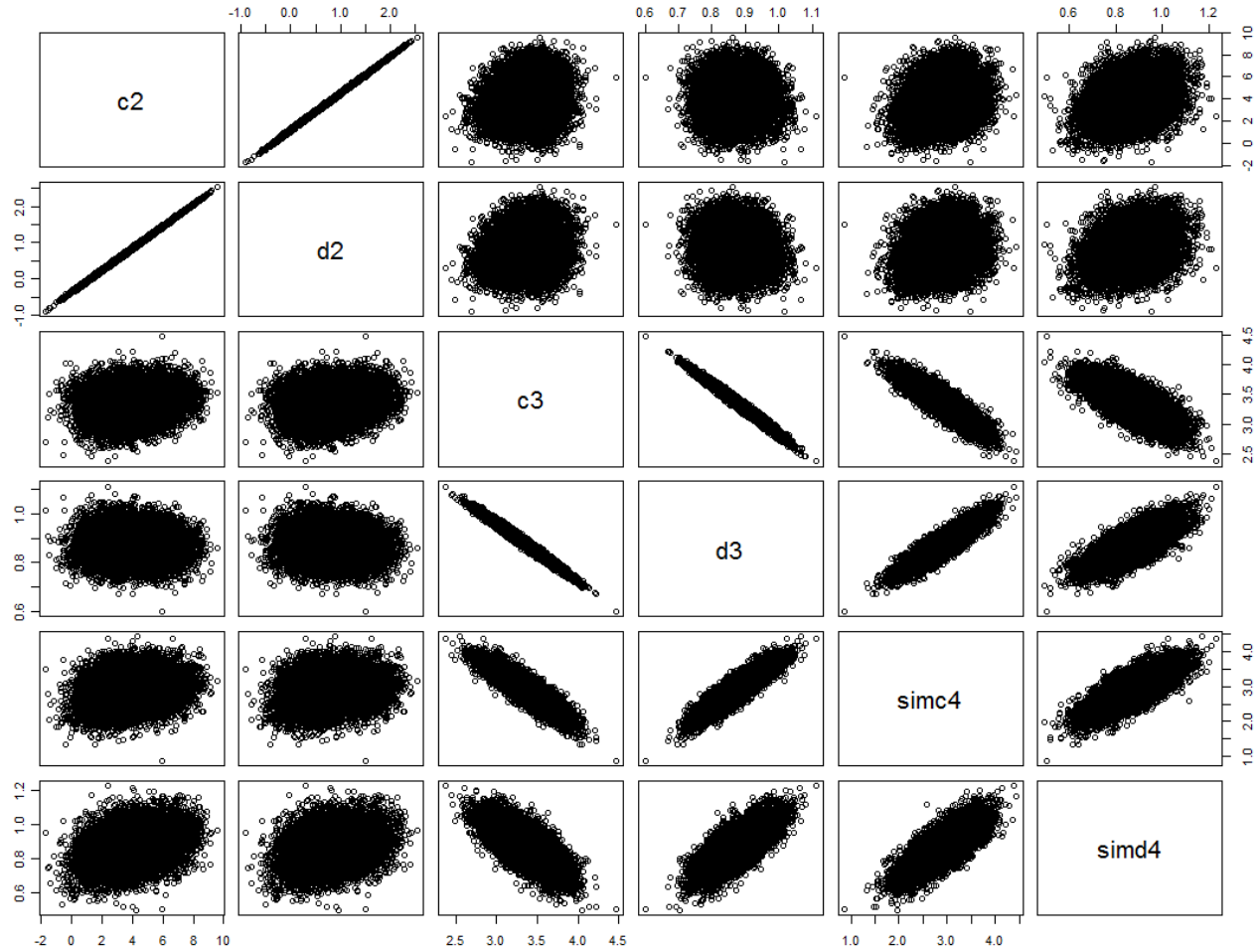
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# Correlation Structure Between Parameter Estimates



# Correlation Structure in First Simulation



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# Model for c4 as function of c2, d2, c3, d3

Call:

```
lm(formula = c4 ~ c2 + d2 + c3 + d3, data = fits)
```

Residuals:

Min	1Q	Median	3Q	Max
-1.01166	-0.46895	-0.07504	0.28905	1.49891

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	3.6953	0.6501	5.684	2.02e-06	***
c2	0.1758	0.3269	0.538	0.5941	
d2	-0.9903	1.0051	-0.985	0.3313	
c3	-0.4872	0.2456	-1.984	0.0552	.
d3	1.4738	0.7490	1.968	0.0571	.

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6164 on 35 degrees of freedom

Multiple R-squared: 0.2295, Adjusted R-squared: 0.1414

F-statistic: 2.606 on 4 and 35 DF, p-value: 0.05239

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# Model for d4 as function of c2, d2, c3, d3

Call:

```
lm(formula = d4 ~ c2 + d2 + c3 + d3, data = fits)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.35653	-0.13859	-0.02136	0.12273	0.52678

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	1.00471	0.22184	4.529	6.6e-05	***
c2	0.09111	0.11153	0.817	0.420	
d2	-0.42320	0.34298	-1.234	0.225	
c3	-0.11051	0.08380	-1.319	0.196	
d3	0.36019	0.25558	1.409	0.168	

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2103 on 35 degrees of freedom

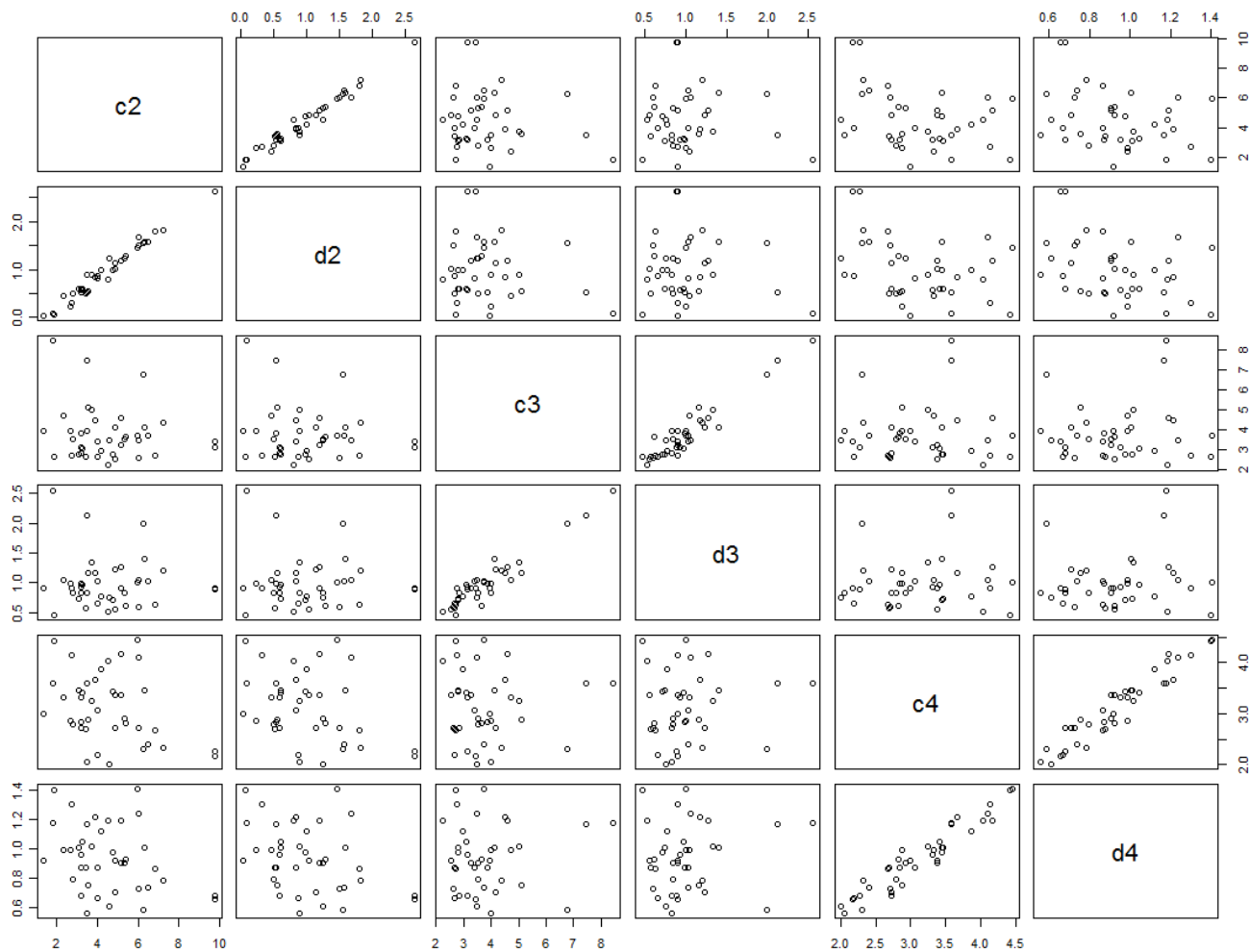
Multiple R-squared: 0.2004, Adjusted R-squared: 0.109

F-statistic: 2.193 on 4 and 35 DF, p-value: 0.09

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# Correlation Structure in Second Simulation



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# Distributions of Crack Lengths

