Demonstrations of Model-assisted Probability of Detection (MAPOD) Evaluation

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Case Numbers: 88ABW-2011-4699, 88ABW-2011-4701, 88ABW-2011-4117



What is Probability of Detection (POD) and Model Assisted POD (MAPOD)?

• Probability of detection (POD) of a certain discontinuity as a function of some size metric given a defined inspection technique and target population.



• We define "MAPOD" as the collection of approaches that *use models* of inspections as some portion of the inputs that are processed to yield an *estimate of POD*.



Model-Assisted POD Model Building Process [MIL-HNBK 1823A, Appendix H (2009)]



Develop Complete Approach to Uncertainty Propagation in MAPOD



Evaluating Reliability Using Simulated and Empirical Data:

- To mitigate cost of validation study, one must better assess the critical sources of error and variation on reliability performance
- Hoppe [2009] presented historical case highlighting benefit of improving the measurement model through including crack length and depth in fit

•
$$\hat{a} = \beta_0 + \beta_1 a_1 + \varepsilon$$

•
$$\hat{a} = \beta_0 + \beta_1 a_1 + \beta_2 a_2 + \varepsilon$$

 $\hat{a} = \beta_0 + \beta_1 f(a_1, a_2) + \varepsilon$



Objective: Explore Case Studies to Assess Impact of Measurement Model Quality on POD Estimation and Sample Number



'Progress' on Model-assisted POD Evaluation (Highlights of Talk)

- Leverage *physics-based models* in POD evaluation that represent *key factors* in an NDE technique: ⁻⁻⁻⁻⁻
 - Present status on models to address real cracks in material noise (EC, UT)
 - Assess benefit of <u>more accurate models</u> on POD fit and <u>sample requirements</u>
- Present *analysis tools* to address propagation of model error, uncertainty in calibration and Bayesian refinement [TRI/Austin]
- Present demonstration of MAPOD protocol for SHM (in-situ NDE) validation [Radiance]
 - Address full spectrum of environmental conditions
 - Consider independence of each inspection site
 - Evaluate POD as function of sensor system durability (time)



Demonstration: Eddy Current Inspection of Surface-breaking Cracks in Ti-6AI-4V



- tilt
- dimensions, windings (probe to probe variability)
- Scan resolution (fixed)
- DAQ hardware (Agilent Impedance Analyzer, Nortec 19ell)
- Calibration process
- Human data interpretation

- isolate liftoff direction in response
- set full screen height for known notch (0.10")
- use quantitative metrics in evaluation
- 50 100 150 200

Comparing Cracks and EDM Notch – Experimental and Simulated Data

Compare Cracks and EDM Notch Responses:

- Largest cracks very similar in reactance (dX)
- Lower response [€]
 in resistance (dR), due to 'difference' in width of cracks and notches at surface
- Differences for smallest crack (0.040") may be due to crack closure effect and/or uncertainty in aspect ratio



Comparing Cracks and EDM Notch – Experimental and Simulated Data

Compare Agilent and Nortec 19ell Responses:

- Same probe @ 2 MHz was tested over crack specimens [Cherry]
- Signal to noise much improved for Nortec 19ell wrt impedance results
- Increase signal to noise due to signal conditioning (zero response over specimen and scaling of signal)
- Background noise seems to be more associated with measurement noise than actual material noise (surface roughness, grain noise)





Simulating Cracks in Material Noise in VIC-3D[©]

- Material noise model (anisotropy, grain structure) can simulate random variation of material parameters for a volume grid (flaw region)
- Material Parameters Volume Fraction Viewer Example: randomize Conductivity View/Edit Cell-Z=1 Z=2 3E7 R S/m conductivity of material × Ð Y. 1 Permeability (grains) 1 Z: Rel. 1 ¥ Value: 0.24421 Permittivity Optimize Configure Per-Cell Material Randomness R Rel. 1 Y Material Parameters Per-Cell Material Randomness Conductivity Permeability Magnetic Loss O Permittivity Random Distribution Ohm/m 0 R ¥ Magnetic Loss Uniform Gaussian Minimum: Conductivity, S. 🗸 2.9003E7 Standard Deviation Randomize Per Cell Edit... Maximum: 0 1e6 Conductivity, S. 🗸 Conductivity: Permittivity: - X Axis + 3.0983E7 0 0 Permeability: Magnetic Losses: Grid Shape Parameters Position of Center of Grid in Workpiece × 1 Width, mm X position, mm ✓ 0 ✓ 2 Length, mm Y position, mm ✓ 0 OK. Heip Cancel Height, mm ✓ 3 Z position, mm **v** 0 0K Cancel Help

Multiscale flaw model enables simulation of cracks (notches) at boundaries (edges, holes) and in the presence of surface roughness, grain structures [see Murphy et al, "Advances in Developing Multiscale Flaw Models for Eddy-Current NDE", THUR. 1:30] [see Sabbagh et al, "Characterizing Randomly Anisotropic Surfaces", THUR. 2:00]

Weasurement 'Model' and POD Evaluation

←

Input Parameters Types:

- Controlled Parameters, $a_i(N_i)$
 - Flaw size
 - Flaw location
 - Temperature Conditions
 - Ambient noise
- Uncontrolled Parameters, $a_k(N_k)$
 - Boundary conditions
 - Flaw morphology

Input Parameter Characteristics:

- Expected Variation Represented as a *Distributions* (ex. *Gaussian*, Uniform, Gamma, Beta)
- Uncertainty in Distribution
 Parameters (Not Addressed)

Level 1. Input Parameter Variability



2

length

3







200

0

0





Measurement 'Model' and POD Evaluation



POD/MAPOD Toolkit (TRI/Austin) -Features and Interface Requirements

Perform POD Assessment and Provide Diagnostic Tools:

- Define Model Input Parameters
- Manage Multiple Data Sets
- Select Model Parameter Fitting Approach
- Perform POD Assessment and Provide Diagnostic Tools

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- POD Toolkit component of NDI Toolbox (open source)
- POD Toolkit contains code with example case studies, and directory structure for code, data and temp. files
 - ex: hitmiss, ahat_vs_a, ahat_vs_a1a2, bayesmcmc,
 - mapod demos
- .cfg files: Define POD/MAPOD Model Structure (Construct model tree)
- Python code: Adjust interface for Unique POD / MAPOD Evaluation Features
- *R* code: Perform statistical evaluation
 - Optional R link to WinBUGS for Bayesian Analysis

Simulated POD Studies: EC Inspection of Cracks in Ti-6AI-4V

Input Parameters in Study:

- a₁ = Crack length
 - primary variable for POD
- $a_2 = Crack depth (width)$
 - dependent variable on a1 crack length
 - relationship define by function a2(a1) = a4 * a1
 - a4 is the aspect ratio and defined as an random variable
- a₃ = Liftoff
 - uncontrolled parameter during study
 - estimation of liftoff
 could improve POD
 performance (to verify)



Input Parameters Distributions

Simulated POD Studies: Input Parameter and Model Response

Input Parameters in Study:

- a₁ = Crack length
 - primary variable for POD
- a₂ = Crack depth (width)
 - dependent variable on a1 crack length
 - relationship define by function a2(a1) = a4 * a1
 - a4 is the aspect ratio and defined as an random variable
- $a_3 = Liftoff$
 - uncontrolled parameter during study
 - estimation of liftoff
 could improve POD
 performance (to verify)



EC Response wrt Crack Length (variation for a2 and a3 included)



Simulated POD Studies: One vs. Two Parameter Models

Perform POD Assessment and Provide Diagnostic Tools:

Ex: ahat versus a_1 and a_2 model fit: $\hat{a} = \beta_0 + \beta_1 a_1 + \beta_2 a_2 + \varepsilon$

- Explore: Case study 3: dep_fixed POD(a_1, a_2) generated for varying a_1 and for a dependent variable $a_2(a_1)$ using a deterministic model
 - POD plot over a uniform distribution of a_1
 - $-a_2 = m a_1$ where: *m* is a constant value = 0.33
- Compare different model fit and confidence bounds approaches
 - Analysis 0: Neglect a₂
 - Analysis 1: Regression fit, Delta method for confidence bds (ahat versus a_1 and a_2)
 - Analysis 2: Regression fit,
 Monte Carlo for confidence bds (ahat versus a₁ and a₂)
 - Analysis 2: Bayesian (MCMC) for model fit and confidence bds (ahat versus a_1 and a_2)

		Analysis 0	Analysis 1	Analysis 2	Analysis 3
		survreg()	survreg()	glm()/MC	BayesMCMC
		neglect a2	use a1, a2	use a1, a2	use a1, a2
T	BO	-0.05780	-0.05986	-0.05986	-0.05983
	B1	5.39532	2.77503	2.77503	2.77668
	B2	0.00000	6.65178	6.65178	6.64630
	Delta	0.02538	0.02001	0.02001	0.02061
	Threshold	0.10000	0.10000	0.10000	0.10000
	var11	0.00003	0.00002	0.00002	0.00002
	var22	0.00912	0.11971	0.12346	0.12687
	var33	0.00000	0.73494	0.75790	0.78247
	a50	0.02925	0.03204	0.03204	0.03203
	a90	0.03529	0.03719	0.03721	0.03732
	a90/95	0.03616	0.03720	0.03837	0.03733

- Inclusion of a₂ reduces residual variance (Delta)
- Very little difference observed between the three methods (1-3) for this case

Simulated POD Studies: Use Physicsbased Model, Vary Sample Number

Perform POD Assessment of VIC-3D[©] Model Fit:

- Calibrated Physics-based Model: $\hat{a} = \beta_0 + \beta_1 f(a_1, a_2) + \varepsilon$
- Use mean liftoff from distribution for model
- Vary Number of Samples (N= 100, 50, 25, and 12)
- **Results:** 'ahat-vs.-a' analysis much less sensitive to sample number used (compared to 'hit miss')







samplesz	100	50	25	12
max flaw				
size	0.100"	0.100"	0.100"	0.100"
liftoff	unknown	unknown	unknown	unknown
B0	0.004	0.000	0.006	0.011
B1	0.952	0.985	0.956	0.915
Tau (scale)	0.024	0.019	0.020	0.014
Mu	0.101	0.101	0.098	0.097
Sigma	0.025	0.019	0.021	0.015
Threshold	0.100	0.100	0.100	0.100
a50	0.101	0.101	0.098	0.097
a90	0.132	0.126	0.125	0.117
a90/95	0.138	0.133	0.135	0.127
a90/95 -				
a90	0.006	0.006	0.010	0.010
var11	9.66E-06	1.05E-05	2.56E-05	2.41E-05
var12	-7.08E-07	-5.87E-07	-1.88E-06	-1.22E-06
var22	3.21E-06	3.88E-06	8.97E-06	9.86E-06
SDa90	0.00362	0.00392	0.00596	0.00610



Demonstration: Ultrasonic Inspection of Cracks in Ti-6AI-4V

- Compare: Simulated (CIVA) and Experimental UT of Ti Lugs
- Vary crack length, aspect ratio, angle, crack morphology Results:
- Sensitivity to Crack Size, Aspect Ratio, Angle and Morphology Demonstrated in Exp. and Simulation









Need to Address Morphology for Crack Sizing

Probabilistic Reliability Assessment for SDS Systems



Protocol comprises:

- Procedure for analyzing all pertinent characteristics of the SDS system
 - Identify all **critical factors** that affect system performance

- Multistage approach for system validation

- Modeling and experimental methodology for efficiently addressing a wide range of damage and operational conditions
- Effective methods for evaluating metrics of capability and reliability depending on system type and function (*uncertainty propagation*)

[see papers by Lindgren et al, Aldrin et al, and Medina et al, IWSHM conf. 2011] 20 July 2011

Primary Protocol



Demonstration Study – Define SHM System

SDS System Characteristics:

- **Type:** Direct damage detection using active sensing
- SHM System Output: Damage detection call
- Coverage and Sensor Location: Semi-global (sub-structure)
- Measurement Type: Vibration (low frequency) response
- Time of Data Acquisition (DAQ): While aircraft is on the ground
 - Vary temperature (gradients), loading/unloading, boundary cond., fastener torques
- Location of DAQ Hardware: Onboard the aircraft
- **Structure Characteristics:** Include joints in test article
- Center joint with sites for simulating damage growth
- End conditions with optional shims (to change boundary)
- Damage Characteristics:
- Damage Types (Failure Conditions) to Detect: (Large) fatigue cracks
 - Approximate crack growth by cutting notches
 - Fastener removal necessary for growing flaw (must maintain equal torque, verify damage metric change not due to changes in boundary conditions)







Demonstration Study – Identify and Evaluate Controlling Factors









Temperature Study: Test article placed in Thermotron temperature chamber

- Temperature testing performed from -20°F to 150°F
- Temperature compensation algorithms are necessary for damage metric
- Significant temperature gradients also observed during study
 - Some gradients considered extreme (>45°F) due to end ' thermal sinks'
 - Need to make estimate of expected gradients 'in the field' (10-20°F ?)







Observations:

- Damage grown at 1/16" increments up to 0.688" at only one site to verify sensitivity (thin saw blades provided by NIAR)
- Simulated flaw growth (SFG) attempted to mimic forcing of plate structure without creating damage – no significant effect on damage metric
- Sensitivity observed to certain notch increases, but trend not linear
 - sensitive to first notch cut
 - significant drop after fastener installed and removed (FIR)
 - Metric grows with larger notches
- Jump observed after two week delay in testing – 'still in noise'
- Larger cuts will be applied for validation studies





Demonstration Study: Focused on Single Stage

- Phase II Laboratory Testing of Relevant Structures / Environment
- Assumption: Key SDS Factors can be Demonstrated in Single Study

Factors in Study:

- Flaw growth (notch):
 - First cut: 0.063", Second to 0.125",
 repeat 0.125" cuts to 1.00" (10 levels)
 - At two fastener locations with relief notches
- Environmental conditions: (ambient 72°F)
 - Temperature variation (32°F to 112°F)
 - Temperature gradients (<10°F)
 - Ambient noise (chamber on / off)
- Boundary conditions:
 - Loading / unloading mass on structure (10 lb)
 - Fastener removal and reinstall (75 in-lbs +/- 10 in-lbs) 'simulate maintenance'
- Sensor conditions: Evaluate accelerometer bond reinstallation (ref., second) 20 July 2011





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Fit Measurement 'Model' (Using Empirical Data)

•Flaw length (a_1): $dm = \beta_0 + \beta_1 * a_1 + \beta_2 * a_1^2 + \beta_3 * a_1^3$

•Flaw location (*a*₂)

- Evaluate both 'combined' and 'separate' flaw location scenarios fits
- •Normalized mean **temperature** (a_3) , and absolute value $|a_3|$
- •Normalized temperature gradients (a₄),
- •Abs. difference between temp. and nearest reference (a_5)
- •Ambient noise level (a_6) ,
- •Sensor status (active, failed)
- Level 2: Uncertainty in Model Parameter Estimate



Regression Analysis Example (R)

Code:	data.tmp <- read.csv('analy_ref1_flaw3.csv',header=FALSE) xl <- data.tmp\$V1 x2 <- data.tmp\$V2 x3 <- data.tmp\$V3 x4 <- data.tmp\$V4 x5 <- data.tmp\$V4 x5 <- data.tmp\$V5 x6 <- data.tmp\$V6 x11 <- x1*x1 x11 <- x1*x1						
	y1 <- data.tmp\$V14 frame1 <- data.frame(y=y1, x1=x1, x2=x2, x3=x3, x4=x4, x5=x5, x6=x6, x7 = x11, x8 =						
	X111) $y_1y_2y_3 = (m(formula = y_1, y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8) data=from a1)$						
	summary(v.vs.x) $y = x1 + x2 + x3 + x4 + x3 + x6 + x7 + x6, data=framer)$						
Call:	$lm(formula = y \sim x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8, data = frame1)$						
Residuals:	Min 1Q Median 3Q Max						
	-0.035835 -0.007133 0.001119 0.006437 0.026368						
Coefficients:	Estimate Std. Error t value Pr(> t)						
	(Intercept) 0.018921 0.003902 4.849 6.8e-06 ***						
	x1 -0.081361 0.041766 -1.948 0.05526 .						
	x2 -0.005525 0.003405 -0.959 0.34072 x2 0.010200 0.002600 2.704 0.00665 **						
	x4 0.000321 0.005813 1.603 0.11315						
	x5 0.032816 0.010755 3.051 0.00318 **						
	x6 0.005763 0.013645 0.422 0.67402						
	x7 0.373822 0.109798 3.405 0.00108 **						
	x8 -0.205131 0.072407 -2.833 0.00596 **						
	Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1						
Diagnostics:	Residual standard error: 0.01303 on 73 degrees of freedom						
	Multiple R-squared: 0.9133, Adjusted R-squared: 0.9037						
	F-statistic: 96.07 on 8 and 73 DF, p-value: < 2.2e-16						
Significant	 x1 <- data.tmp\$V1: Flaw size (a1) (Part of flaw size model) 						
Factors:	 x3 <- data.tmp\$V3: Normalized mean temperature (a₃) 						
	 x5 <- data.tmp\$V5: Normalized temperature gradients (a4), 						
	• $x7 <-x11 <-x1*x1$ Flaw size model term: $(a_1)^2$						
	• $x8 <-x111 <-x1*x11$ Flaw size model term: $(a_1)^3$						









POD Results: Dependency on Flaw Location



Can Improve POD by Choosing *Optimal Sensor Configuration*:



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- **POD Evaluation Must Address Known Sensor Durability Issues:**
- Issue demonstrated by percent of C–17 in-service strain gauge failures as a function of time [Ware et al] ---->
 ^{0.50}
 [0.45]
- Bathtub Curve Model [Meeker and Escobar]



- **Evaluation of Impact of Sensor Failure:**
- Evaluate changes in POD due to random sensor failure over time
- Explore failure of two sensors (25%) over first six years of service life
- Distributions of Time to Failure Considered in Evaluation
 20 July 2011



time (years)



POD Results – Impact of Sensor Durability



Sensor Scenarios with
 Corresponding Changes in
 POD and False Call Rate:

Scenarios Addressing Sensor Failure Approach 1: (Best Sensitivity)

- Use accel. #1 as reference
- Use accel. #6 as source



0.2

Approach 2: (Accel.#6 Failure)

- Use accel. #1 as reference
- Use median of active sensors



- Use median of active sensors



0.5 a₁ (in.)

Approach 4: (Accel.#8 Failure)

- Use accel. #3 as reference
- Use median of active sensors



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POD Results – Impact of Sensor Durability



Evaluation of Impact of Sensor Failure:

- Evaluate changes in POD due to random sensor failures over time
- Distributions of Time to Failure Considered in Evaluation
- Results: Mean expected POD and POFC at a flaw size of 1.0 in as a function of time







- Progress on MAPOD for NDE and SHM:
 - **Protocol:** NDE \rightarrow MH1823A; SHM \rightarrow Aldrin et al 2011
 - Tools: Validated Models, MAPOD Toolkit [TRI/Austin]
- Key Insight from EC / UT MAPOD Demonstrations
 - Models in evaluation has potential to impact sample requirements
 - Crack morphology is a significant factor in NDE measurements
 - Challenges exist to quantify complete source of noise, error
 - Identify limits on purely models assisted approaches

Key Insight from SHM MAPOD Demonstration

- Must ensure changes in SHM metric are truly damage growth
- Certain flaw locations may require separate POD models
- Feasible to evaluate impact of sensor failures on performance
- Need MAPOD approach to cover all damage scenarios, over time
- Need better understanding of variabilities through empirical data collection, forward and inverse modeling



- Air Force Research Laboratory NDE Branch (RXLP) Jason Heebl, Josh Shearer, John Brausch, Rick Reibel, Mark Blodgett
- University of Dayton Research Institute (UDRI): Matt Cherry, Ryan Mooers
- Victor Technologies Elias Sabbagh, R. Kim Murphy
- Radiance Technologies
- Jose Santiago Univ. of Florida
- TRI/Austin: Chris Coughlin
- CNDE Iowa State University
- Irving Gray NDE Technologies
- Floyd Spencer SfHire
- Charles Annis Statistical Engineering
- The R software environment for statistical computing and graphics was used for all statistical computation and statistical plots. R is open-source (free) software and is available for download here: <u>www.r-project.org</u>



- I. Modeling and Simulation:
- Model Benchmarking and Validation
 - Error analysis for uncertainty propagation
 - Provide insight to modeling gaps and needs
- Forward Model Development Efforts (UT, EC, IR)
 - Address gaps through development programs
 - Improve simulation time (leverage parallel processing/GPUs)
- Stochastic Modeling
 - Probabilistic Collocation Method
 - Many Unknowns (2D/3D Problems)



- II. Analysis:
- Address Variability and Uncertainty in Input Parameters
 - Statistical Uncertainty Evaluation and Propagation
 - Use of Inverse Methods to Quantify Variability and Reduce Uncertainty
- Develop Comprehensive Approach to A-hat vs A Analysis Using Different Model Types
 - Empirical fit (statistical model)
 - Calibrated numerical model
 - Calibrated numerical model with inverse methods (to estimate uncontrolled parameters)
 - Transfer function approach
- Second Order Probability Analysis
- Merging Empirical and Simulated Results (Bayesian Methods, Diagnostics)



- **III. Extensions and Validation of Process**
- Model-assisted Measurement System Characterization
 - Validation of Localization and Sizing Estimates
 - Phase I SBIR 2011-12 Efforts
- Comprehensive Validation Studies of MAPOD Process