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Uncertainty in Material Characterization Using the Hessian and Filled Waveguide Method

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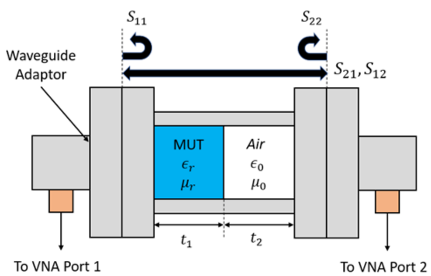
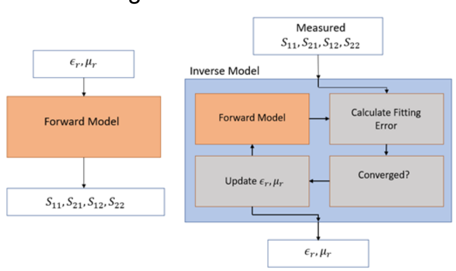
Objective: Develop a method to predict uncertainty in material properties (ϵ_r, μ_r, and thickness) that are calculated using microwave material characterization technique.

Background

- Material properties in the microwave regime are commonly modeled using relative complex permittivity ($\epsilon_r = \epsilon_r' - j\epsilon_r''$) and relative complex permeability ($\mu_r = \mu_r' - j\mu_r''$) .
- The filled waveguide technique is commonly used for microwave material characterization.



- The material properties are obtained from measured S-Parameters using a model of the filled waveguide and an optimization algorithm.



- The filled waveguide is modeled using ABCD parameters which can be cascaded for an arbitrary number of layers.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_i = \begin{bmatrix} \cosh(\gamma_i t_i) & Z_i \sinh(\gamma_i t_i) \\ \frac{1}{Z_i}\sinh(\gamma_i t_i) & \cosh(\gamma_i t_i) \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_i \begin{bmatrix} A & B \\ C & D \end{bmatrix}_i$$

Formulation

- The Jacobian of the forward model represents changes in the S-parameters relative to changes in complex permittivity (or permeability) when evaluated at the material property values of interest. Using the definition of the Jacobian, the Hessian can be calculated as:

$$\mathrm{H}_{\mathrm{fwd}} = J^T J$$

- To evaluate the uncertainty in the material properties, the Hessian for the inverse system is evaluated as follows:

$$\mathrm{H}_{\mathrm{inv}} = (J^T J)^{-1}$$

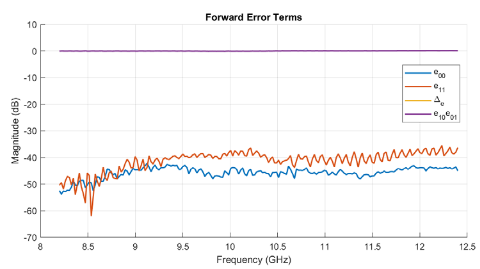
- An estimate of the uncertainty can then be found using the Hessian when scaled by the variance of the measured S-Parameters.

$$U = \sqrt{H_{inv} \cdot (S - Parameter\ Variance)}$$

S-Parameter Variance Sources

- Calibration error
- Flange misalignment
- Imperfect connectors
- Sample inhomogeneity
- Thickness variation
- Sample placement

- The calibration error was experimentally obtained by performing measurements of the calibration standards, using two separate calibration methods.

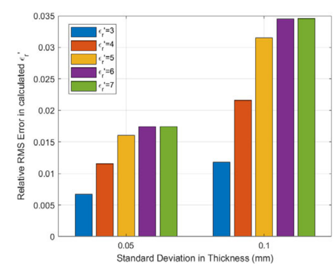


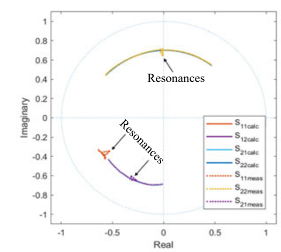
Measurement

- Performed filled waveguide measurements on several materials with well known material properties to evaluate the model.
- After characterizing the materials, the difference between the calculated and listed material properties were compared to the uncertainty value obtained from the model.

Material	Thickness (mm)	Listed Permittivity	Calculated Permittivity	Calculated Uncertainty	Characterization Error
Air	7.5	1.00-0j	1.006-j0.00	±0.0106±j0.0106	-0.006-j0.0
Rogers 3003	1.52	3.00-j0.033	2.968-j0.009	±0.0586±j0.0586	+0.032-j0.024
Rogers 3006	1.28	6.15-j0.012	7.005-j0.033	±0.1108±j0.1108	-0.855+j0.021
Rogers 3010	1.26	10.2-j0.02	10.975-j0.063	±0.2138±j0.2138	-0.775+j0.043
Teflon 1	3.11	2.02-j0.001	1.950-j0.005	±0.031±j0.0310	+0.07+j0.004
Teflon 2	6.78	2.02-j0.001	1.996-j0.007	±0.0257±j0.0257	+0.024+j0.006

- The S-Parameter Variance used in calculating the uncertainty did not include the effect of several sources of variance.
- For example, uncertainty in the measured sample thickness can result in additional uncertainty in the material properties.
- The current forward model is unable to model the resonances that arise from imperfect measurement fixture.





Conclusion and Future Work

- The calculated uncertainty values help give an estimate of the uncertainty in the calculated parameters using the inverse model.
- This uncertainty estimate is not entirely accurate as it does not yet account for several potential errors in the measurements.
- Future work will focus on including all possible sources of error in addition to applying this method to other material characterization techniques.

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