POD Tutorial Part I I
Review of ahat versus a Strategies

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Overview

- $\hat{a}$ versus *a* data
- Bolt hole example
- Distributions of signals and noise
- Statistical model for the $\hat{a}$ signal
- POD from the $\hat{a}$ versus *a* regression model
- ROC
- MINITAB example

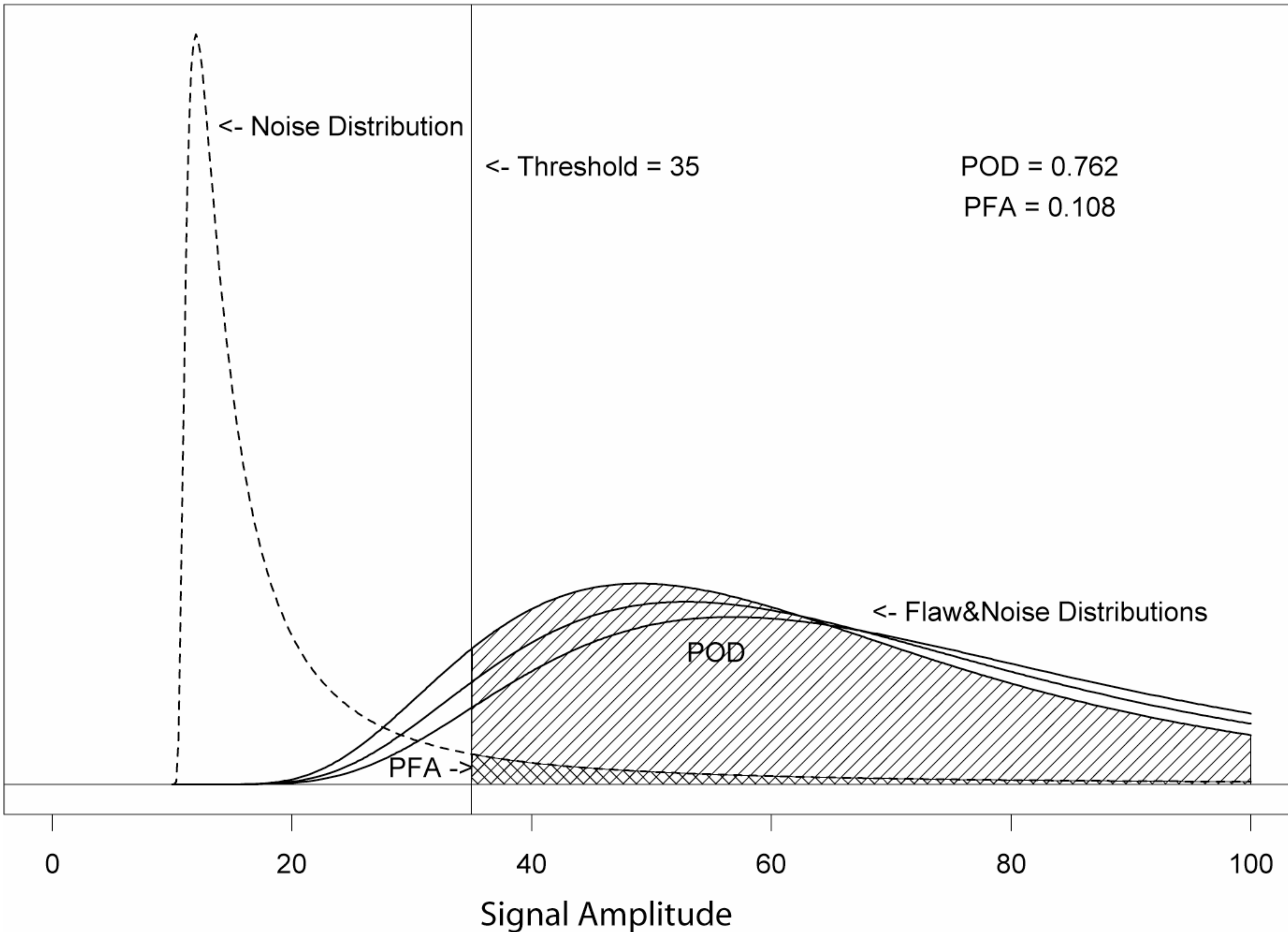
$\hat{a}$ versus a Data

- Each inspection on a flaw gives an indication of signal strength (e.g. % full screen height on an oscilloscope)
- Find a statistical model to describe the distribution of signal strength as a function of flaw characteristics (especially size).
- We use $\hat{a}$ to denote the signal response
- We use a to denote the true flaw size (e.g., crack length or flaw area)

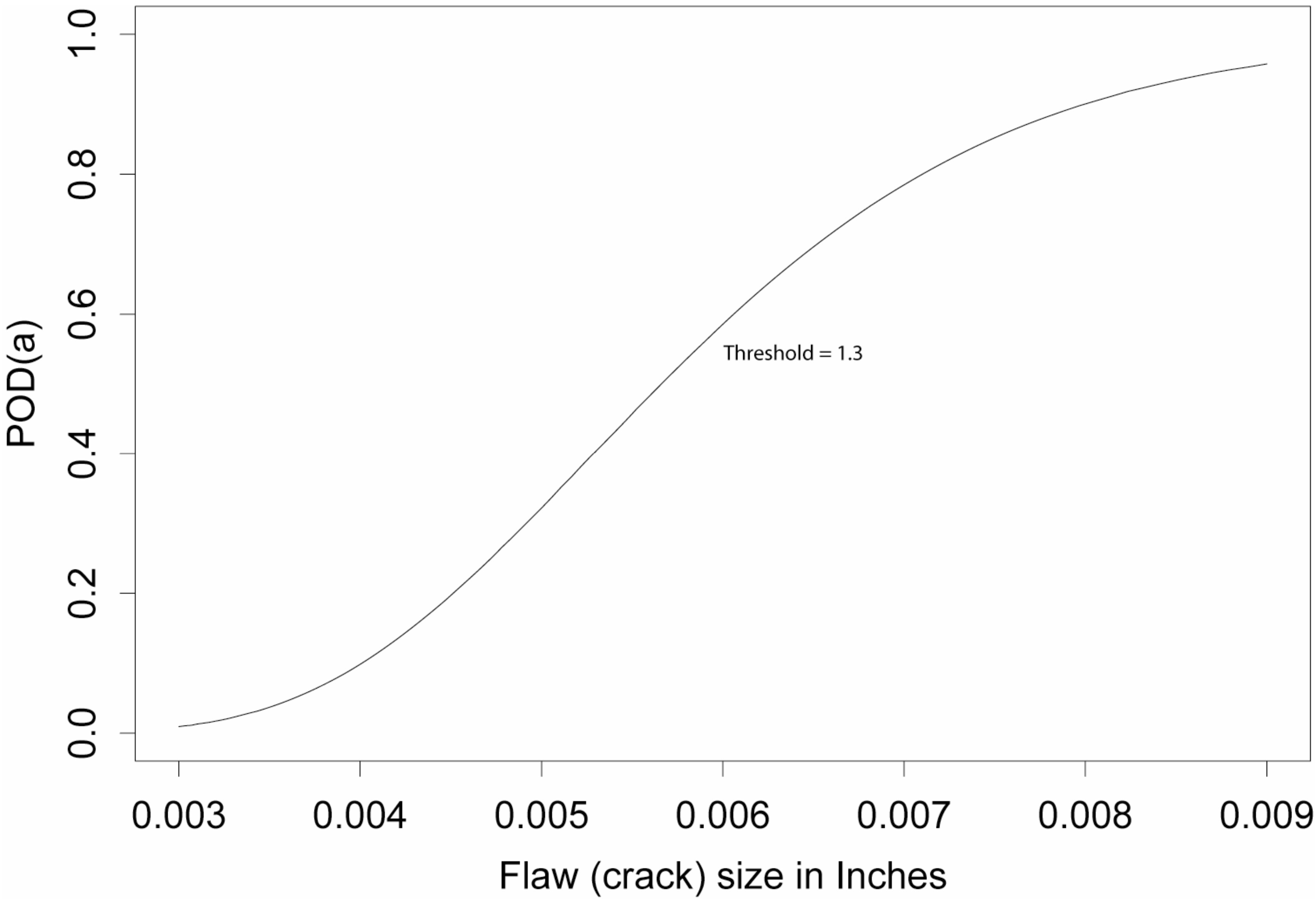
Bolt Hole $\hat{a}$ versus a Data (from MIL-HDBK 1823)

a	$\hat{a}$	a	$\hat{a}$	a	$\hat{a}$
0.001	<1	0.012	2.2	0.022	7.7
0.004	<1	0.012	3.4	0.023	11.6
0.005	1.5	0.012	2.4	0.023	8.0
0.006	<1	0.015	3.0	0.028	>20
0.006	1.2	0.016	7.3	0.029	>20
0.006	2.6	0.018	7.3	0.030	13.2
0.008	1.2	0.018	4.0	0.034	19.6

Noise and Signal Distributions



POD with Threshold 1.3



Key References

- MIL-HDBK-1823 (1999), *Non-Destructive Evaluation System Reliability Assessment*.

- Meeker, W. Q. and Escobar, L. A. (1998), *Statistical Methods for Reliability Data*, John Wiley and Sons, New York.

- R. B. Thompson, W.Q. Meeker, M. Keller, J. Umbach, C.P. Chiou, Y. Wang, R. Burkel, W. Hassan, K. Smith, T. Patton, and L. Brasche, *Update of Default Probability of Detection Curves for the Ultrasonic Detection of Hard-Alpha Inclusions in Titanium Alloy Billets*, report in preparation for the FAA William J. Hughes Technical Center, Atlantic City, NJ

MIL-HDBK 1823

- Written by a group of POD experts in the 1980's. Became official in 1999.
- Covers design of studies, data analysis methods, and methods to estimate POD
  - Hit-miss data
  - $\hat{a}$ versus a data
- Presently being revised by Chuck Annis

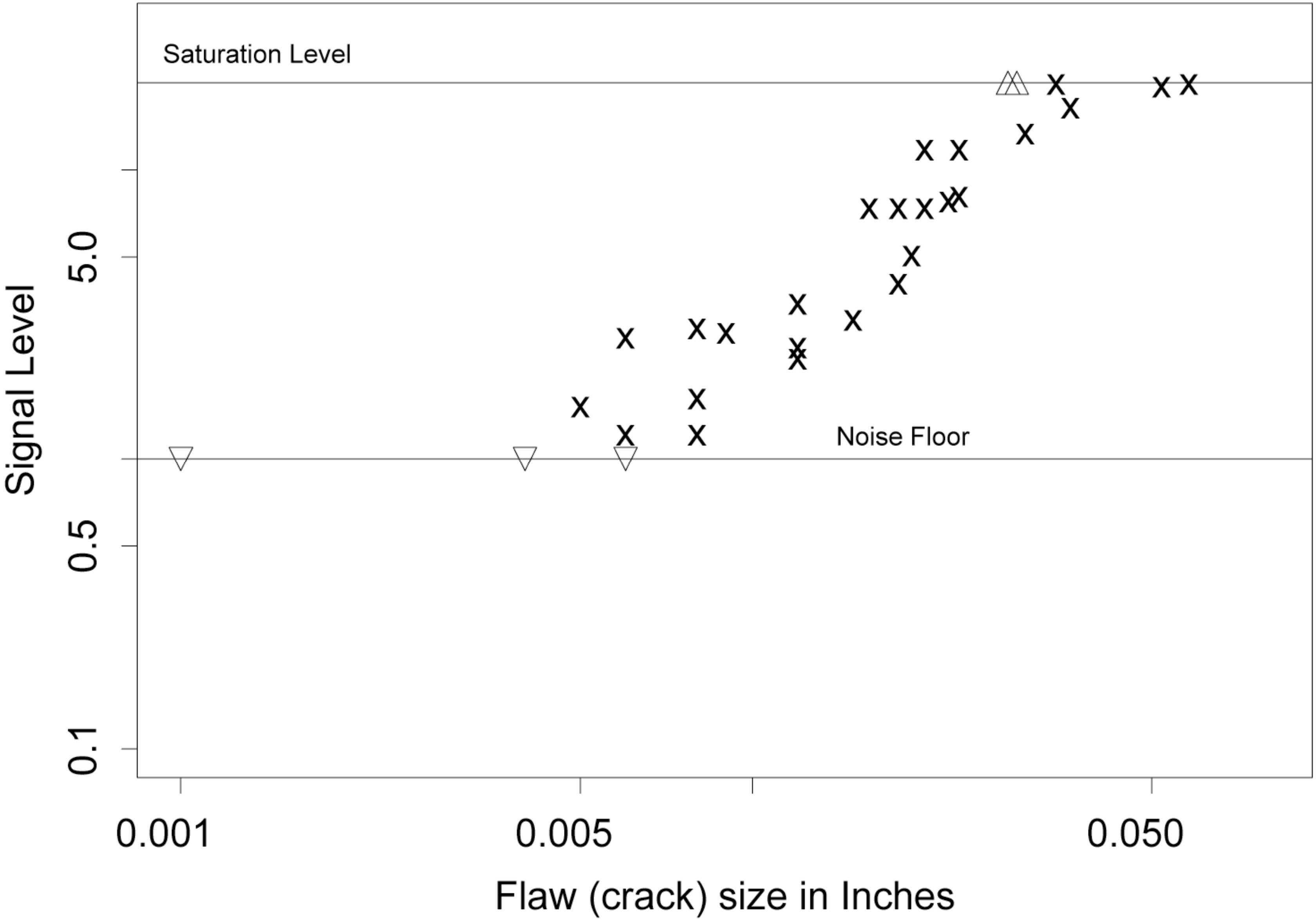
Analysis of $\hat{a}$ versus a Data

- Simple linear regression model
- Some observations may be censored (in which case special software is needed---e.g., MINITAB or JMP)

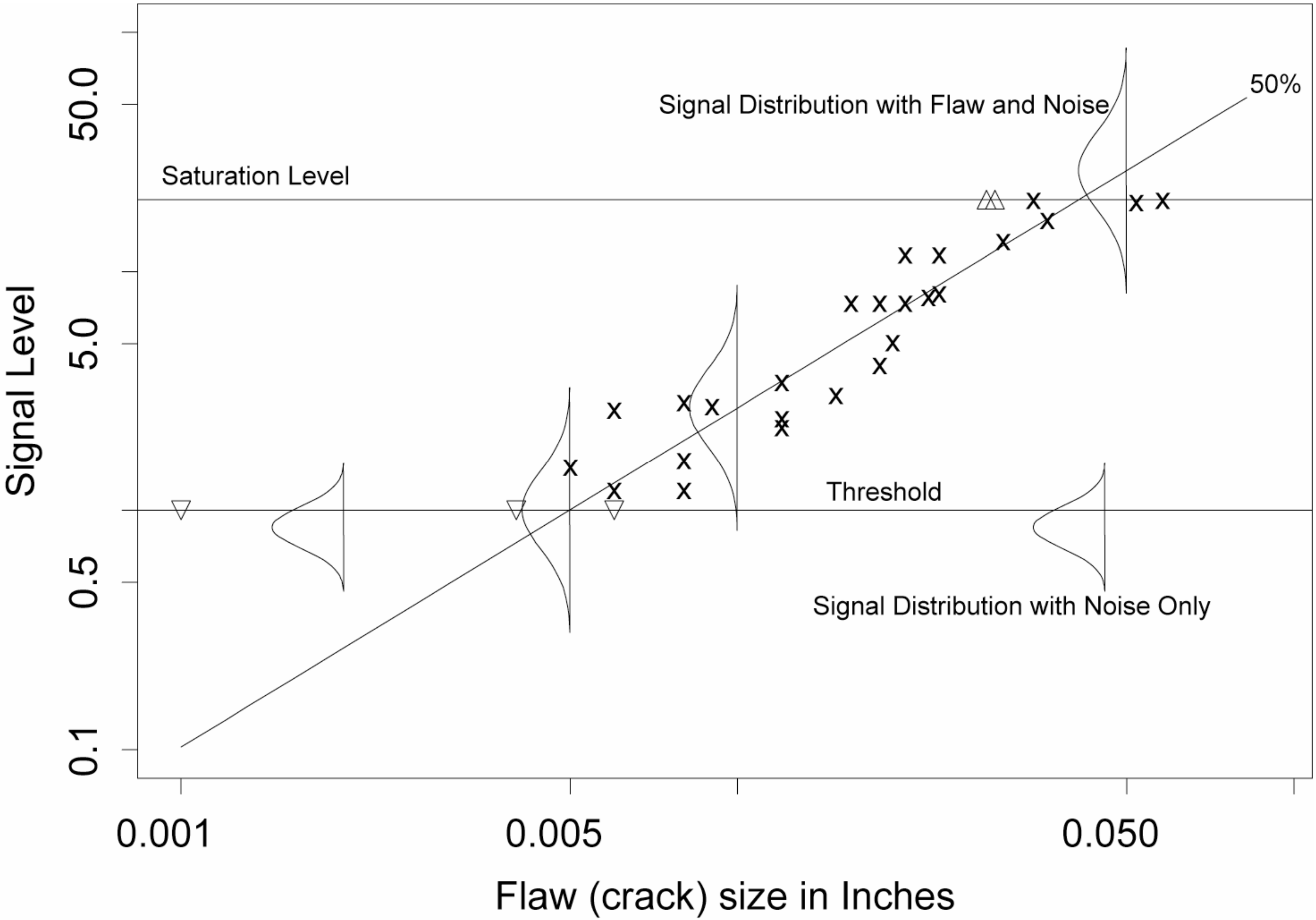
Bolthole $\hat{a}$ versus a Data

Size	Signal	Status
0.001	1	Left
0.004	1	Left
0.005	1.5	Exact
0.006	1	Left
0.006	1.2	Exact
0.006	2.6	Exact
0.008	1.2	Exact
0.012	2.2	Exact
0.023	11.6	Exact
0.023	8	Exact
0.028	20	Right
0.029	20	Right
0.03	13.2	Exact
0.034	19.6	Exact

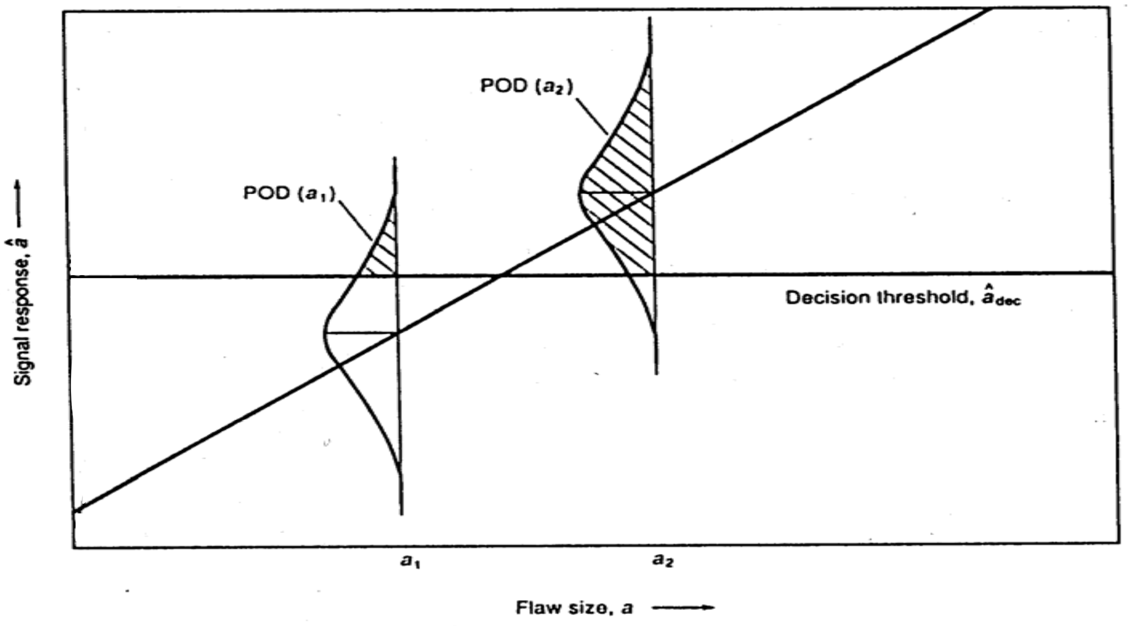
Bolthole Data



Bolthole Data with Threshold and Saturation Level



PICTORIAL REPRESENTATION OF POD DETERMINATION



When distribution is normal, POD for a given flaw size "*a*" is determined by:

- Mean of log amplitude
- Standard deviation of log amplitude
- Decision threshold

The $\hat{a}$ versus a Regression Model

- $\hat{a}$ is used to denote the observed signal (e.g. measured amplitude).
- The simple linear regression model can be expressed as:

$$\Pr(\log(\mathrm{Signal}) < \mathrm{y}) = \Phi\left(\frac{\log(\mathrm{y}) - \mu}{\sigma}\right)$$

$$\mu = \beta_0 + \beta_1 \log(a)$$

σ is constant (does not depend on flaw size a)

Probability of Detection from the $\hat{a}$ versus a Regression Model (1823 notation)

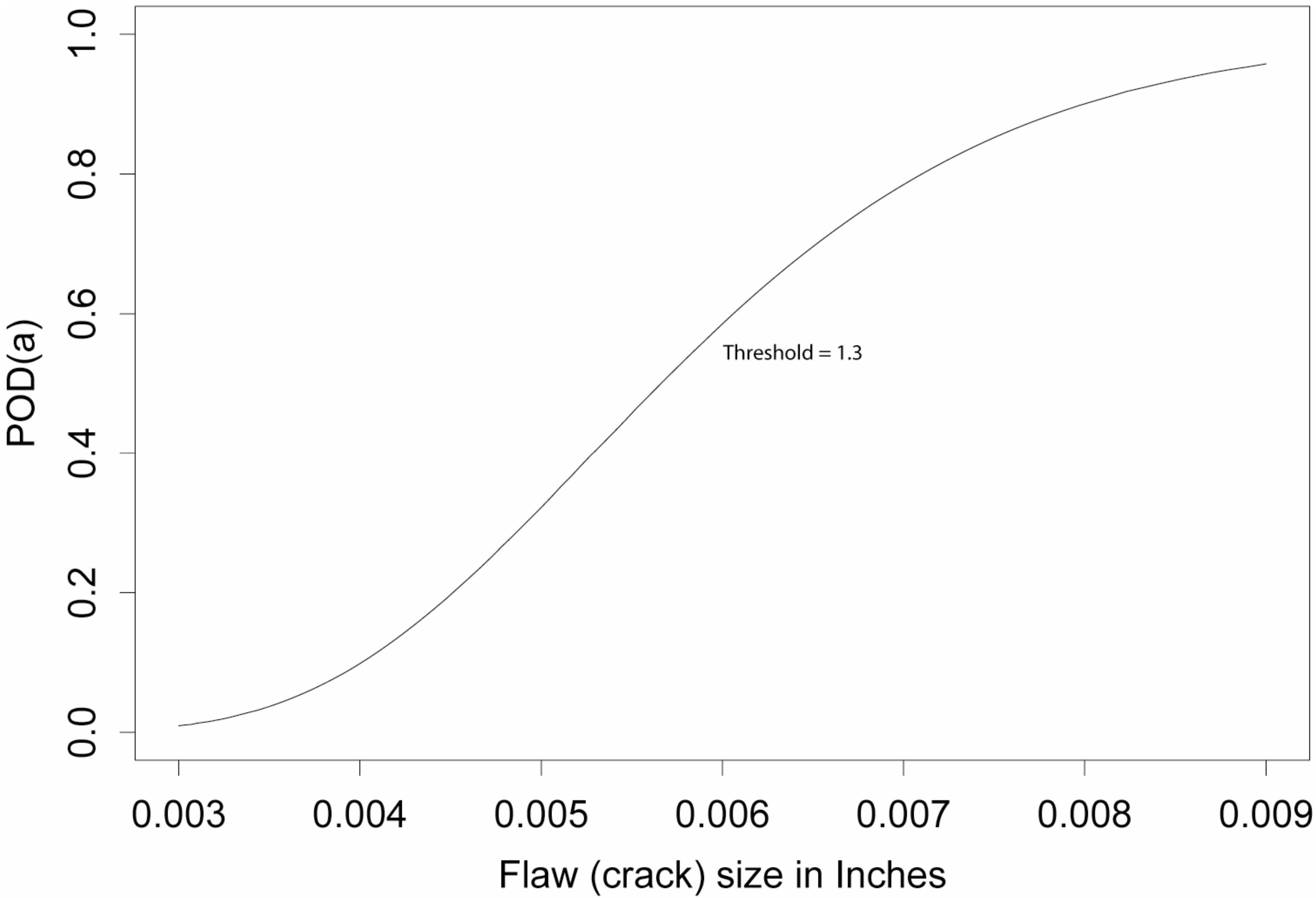
$$
\begin{aligned}
POD(a) &= \text{Pr(Signal} > a_{th}) \\
&= \Pr(Y > \log(a_{th})) \\
&= 1 - \Phi\left(\frac{\log(a_{th}) - (\beta_0 + \beta_1 \log(a))}{\delta} \right) \\
&= \Phi\left(\frac{\log(a) - \mu}{\sigma} \right) \qquad \text{(for symmetric } \Phi\text{)}
\end{aligned}
$$

$$Y = \log(\text{Signal}) = \log(\hat{a})$$

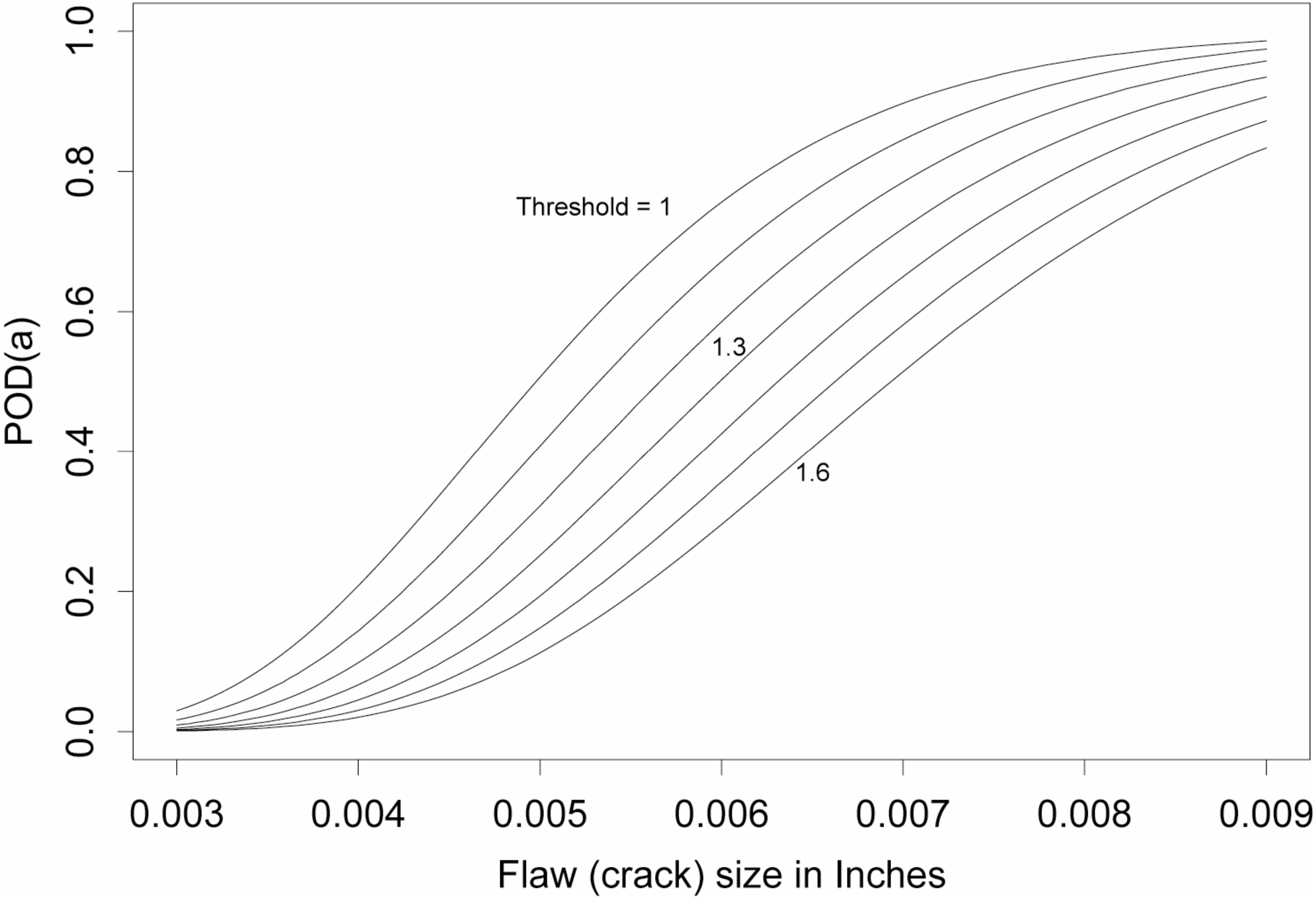
$$\mu = (\log(a_{th}) - \beta_0) / \beta_1$$

$$\sigma = \delta / \beta_1$$

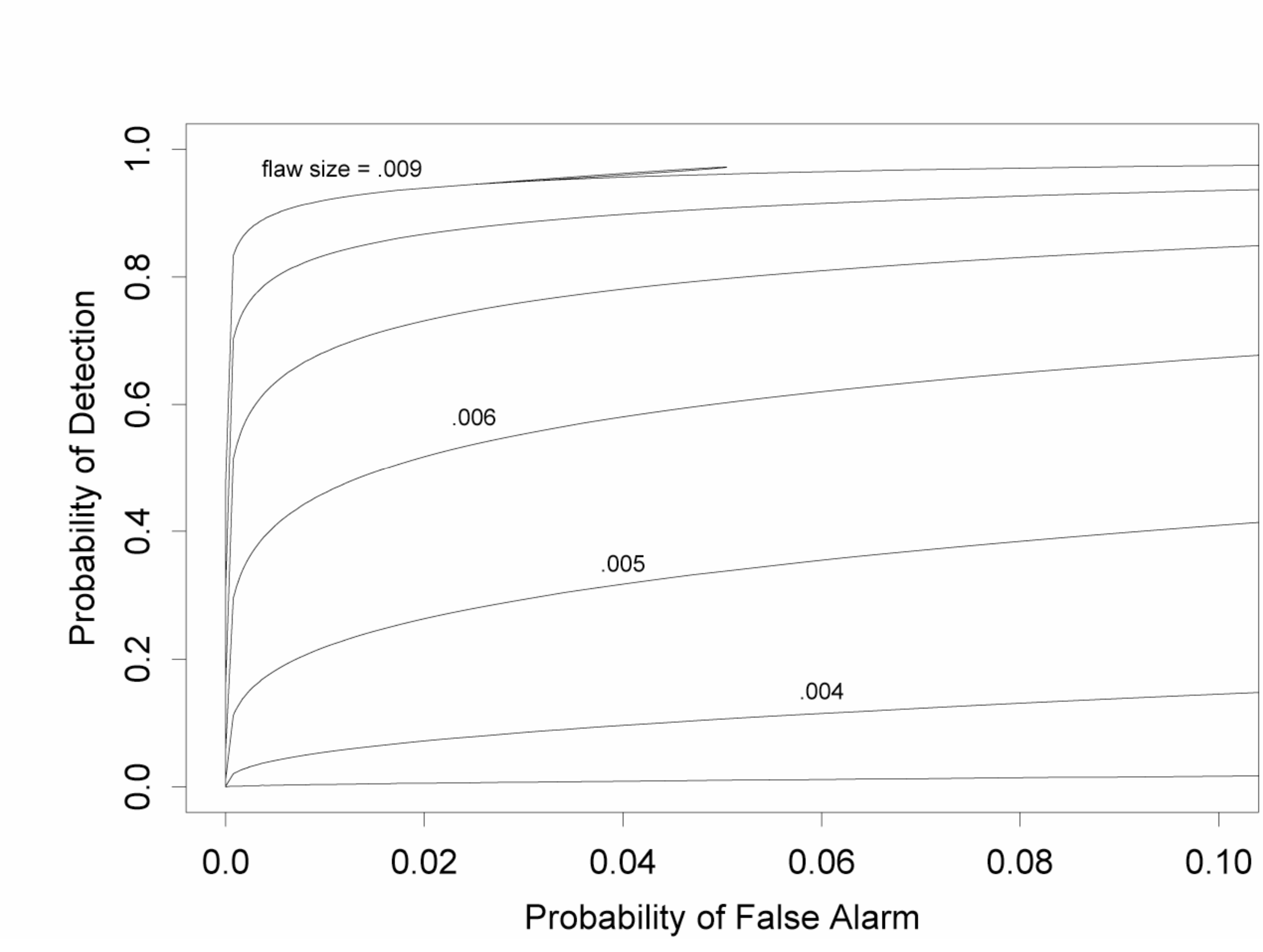
POD with Threshold 1.3



POD with Different Thresholds



Receiver Operating Characteristic (ROC) Curves



MINITAB Estimation of Censored Data Regression Model Parameters

- The MINITAB "Regression with life data" procedure will estimate the parameters β_0 and β_1.
- After the parameters have been estimated, POD (and possibly confidence intervals) could be obtained by
  - Writing a MINITAB macro
  - Importing results to Excel and writing a macro or procedure there
  - Using other simple software (e.g. MATLAB).

MINITAB Worksheet with the Bolthole Data

Size	SignalL	SignalR	Status	Log10Size
0.005	1.5	1.5	Exact	-2.30103
0.001	*	1.0	Left	-3.00000
0.004	*	1.0	Left	-2.39794
0.006	*	1.0	Left	-2.22185
0.006	1.2	1.2	Exact	-2.22185
0.006	2.6	2.6	Exact	-2.22185
0.008	1.2	1.2	Exact	-2.09691
0.012	2.2	2.2	Exact	-1.92082
0.012	3.4	3.4	Exact	-1.92082
0.012	2.4	2.4	Exact	-1.92082
0.023	11.6	11.6	Exact	-1.63827
0.023	8.0	8.0	Exact	-1.63827
0.028	20.0	*	Right	-1.55284
0.029	20.0	*	Right	-1.53760
0.030	13.2	13.2	Exact	-1.52288
0.034	19.6	19.6	Exact	-1.46852

Censored Data Regression Output from MINITAB

Regression with Life Data: SignalL versus LogSize

Response Variable Start: SignalL End: SignalR

Censoring Information	Count
Uncensored value	16
Right censored value	2
Left censored value	3

Estimation Method: Maximum Likelihood

Distribution: Lognormal

Relationship with accelerating variable(s): Linear

Regression Table

Predictor	Coef	Standard Error	Z	P	95.0% Normal CI Lower	95.0% Normal CI Upper
Intercept	**8.38480**	0.684007	12.26	0.000	7.04417	9.72543
Log10Size	**3.72037**	0.366387	10.15	0.000	3.00226	4.43847
Scale	**0.409963**	0.0744166			0.287233	0.585133

Log-Likelihood = -34.735

Probability of Detection from the MINITAB Computer Output

Note and warning: MINITAB ver 14 and other statistical programs use base e (natrual) logs for its lognormal distribution (log of signal)

$$P\hat{O}D(a) = \text{Pr(Signal>}a_{th})$$

$$= \Phi\left(\frac{\log(a)\text{ - }\hat{\mu}}{\hat{\sigma}}\right)$$

$$\hat{\mu} = (\log(a_{th}) - 8.3848)/3.72037$$

$$\hat{\sigma} = \hat{\delta}/\hat{\beta}_1 = 0.409963/3.72037 = 0.11019$$

POD with Threshold 1.3